Homework 9 (optional: won't be graded)

ECS 20 (Winter 2019)

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Exercise 1: 10 points

Using induction, show that $\forall n \in \mathbb{N}, \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$.

Exercise 2: 10 points

Using induction, show that $\forall n \in \mathbb{N}, \sum_{i=1}^{n} i(i+1)(i+2) = \frac{n(n+1)(n+2)(n+3)}{4}$.

Exercise 3: 10 points

Show that $\forall n \in \mathbb{N}, n > 1, \sum_{i=1}^{n} \frac{1}{i^2} < 2 - \frac{1}{n}$.

Exercise 4: 10 points

Show that $\forall n \in \mathbb{N}, n > 3, n^2 - 7n + 12 \geq 0$.

Exercise 5: 10 points

Show that $\forall n \in \mathbb{N}, n > 1$, a set S_n with n elements has $\frac{n(n-1)}{2}$ subsets that contain exactly two elements.

Exercise 6: 10 points

Find the flaw with the following proof that : $P(n) : a^n = 1$ for all non negative integer n, whenever a is a non zero real number:

- *Basis step:* $P(0)$ is true: $a^0 = 1$ is true, by definition of a^0

- *Strong Inductive step:* assume that $a^j = 1$ for all non negative integers j with $j \leq k$. Then note that:

$$a^{k+1} = \frac{a^k a^k}{a^{k-1}} = \frac{1 \times 1}{1} = 1$$

  Therefore $P(k+1)$ is true.

The principle of proof by strong mathematical induction allows us to conclude that $P(n)$ is true for all $n \geq 0$.

Exercise 7: 10 points

Show that $\forall n \in \mathbb{N}$, 21 divides $4^{n+1} + 5^{2n-1}$.

Exercise 8: 10 points

Show that $\forall n \in \mathbb{N} f_1^2 + f_2^2 + \ldots + f_n^2 = f_n f_{n+1}$ where f_n are the Fibonacci numbers.

Exercise 9: 10 points

Show that $\forall n \in \mathbb{N} f_0 - f_1 + f_2 - \ldots - f_{2n-1} + f_{2n} = f_{2n-1} - 1$ where f_n are the Fibonacci numbers.

Extra Credit: 5 points

Show that $\forall n \in \mathbb{N}, n > 1$, a set S_n with n elements has $\frac{n(n-1)(n-2)}{6}$ subsets that contain exactly three elements.