

## Discussion 2: Solutions

ECS 20 (Fall 2016)

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### Exercise 1

Let  $p$  and  $q$  be two propositions.

- a) Write down the truth tables for  $(\neg p \vee q) \wedge q$  and  $(\neg p \wedge q) \vee q$

$p$	$q$	$\neg p$	$\neg p \vee q$	$(\neg p \vee q) \wedge q$	$(\neg p \wedge q)$	$(\neg p \wedge q) \vee q$
T	T	F	T	T	F	T
T	F	F	F	F	F	F
F	T	T	T	T	T	T
F	F	T	T	F	F	F

Based on this result, a creative student concludes that you can always interchange the symbol  $\wedge$  and  $\vee$  without changing the truth table. Is the student right?

The student is right on this specific example. We do not know if it is always true!

- b) Write down the truth tables for  $(\neg p \vee q) \wedge p$  and  $(\neg p \wedge q) \vee p$

$p$	$q$	$\neg p$	$\neg p \vee q$	$(\neg p \vee q) \wedge p$	$(\neg p \wedge q)$	$(\neg p \wedge q) \vee p$
T	T	F	T	T	F	T
T	F	F	F	F	F	T
F	T	T	T	F	T	T
F	F	T	T	F	F	F

What do you think of the rule formulated by the students in (a)?

This is a counterexample that shows that the rule is not always true!

## Exercise 2

Let  $p$ ,  $q$ , and  $r$  be three propositions. Prove or disprove the following statements using truth tables:

a)  $(p \vee \neg p) \wedge (q \vee r)$  is equivalent to  $q \vee r$

$p$	$q$	$r$	$\neg p$	$p \vee \neg p$	$q \vee r$	$(p \vee \neg p) \wedge (q \vee r)$
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	T	T	T
T	F	F	F	T	F	F
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	T	T	T
F	F	F	T	T	F	F

b)  $(p \wedge \neg p) \vee (q \wedge r)$  is equivalent to  $q \wedge r$

$p$	$q$	$r$	$\neg p$	$p \wedge \neg p$	$q \wedge r$	$(p \wedge \neg p) \vee (q \wedge r)$
T	T	T	F	F	T	T
T	T	F	F	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	T	F	T	T
F	T	F	T	F	F	F
F	F	T	T	F	F	F
F	F	F	T	F	F	F

Let us use equivalence instead to prove the two statements:

a)

$$\begin{aligned}
 (p \vee \neg p) \wedge (q \vee r) &\Leftrightarrow T \wedge (q \vee r) \\
 &\Leftrightarrow q \vee r
 \end{aligned}$$

b)

$$\begin{aligned}
 (p \wedge \neg p) \vee (q \wedge r) &\Leftrightarrow F \vee (q \wedge r) \\
 &\Leftrightarrow q \wedge r
 \end{aligned}$$

### Exercise 3

Let  $p$  and  $q$  be two propositions.

- a) Write down the truth table for  $p \oplus \neg q$ .

$p$	$q$	$\neg q$	$p \oplus \neg q$
T	T	F	T
T	F	T	F
F	T	F	F
F	F	T	T

- a) Write down the truth table for  $\neg p \oplus \neg q$ .

$p$	$q$	$\neg p$	$\neg q$	$\neg p \oplus \neg q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	F

### Exercise 4

This exercise relates to the inhabitants of the island of knights and knaves created by Smullyan, where knights always tell the truth and knaves always lie. You encounter two people, A and B. Determine, if possible, what A and B are if they address you in the way described. If you cannot determine what these two people are, can you draw any conclusions?

- a) A says “At least one of us is a knave” and B says nothing.

We proceed as in class. We check all possible “values” for A and B, as well as the veracity of their statements:

Line number	A	B	A says “At least one of us is a knave”
1	Knight	Knight	F
2	Knight	Knave	T
3	Knave	Knight	T
4	Knave	Knave	T

We can eliminate:

- Line 1, as A would be a knight but he lies

- Line 3 and 4, as A would be a knave, but he says the truth

Line 2 is valid, and it is the only one. Therefore, A is a knight and B is a knave.

b) A says “We are both knaves” and B says nothing.

Line number	A	B	A says “We are both knaves”
1	Knight	Knight	F
2	Knight	Knave	F
3	Knave	Knight	F
4	Knave	Knave	T

We can eliminate:

- Line 1 and 2, as A would be a knight but he lies
- Line 4, as A would be a knave, but he says the truth

Line 3 is valid, and it is the only one. Therefore, A is knave and B is a knight.

## Exercise 5

This exercise relates again to the inhabitants of the island created by Smullyan, except that this time there are three types of inhabitants, knights that always tell the truth, knaves that always lie, and spies that may tell the truth or lie. You encounter three people, A, B, and C and you know that one is a knight, one is a knave, and one is a spy. Determine, if possible, what A, B, and C are if they address you in the way described. If you cannot determine what these three people are, can you draw any conclusions?

PA: A says “C is a knave”

PB: B says “A is the knight”

PC: C says “I am the spy”

A	B	C	PA	PB	PC	Check
Knight	Knave	Spy	F	T	T	No: A would be a knight that lies
Knight	Spy	Knave	T	T	F	All answers are compatible
Knave	Knight	Spy	F	F	T	No: B would be a knight that lies
Knave	Spy	Knight	F	F	F	No: C would be a knight that lies
Spy	Knight	Knave	T	F	F	No: B would be a knight that lies
Spy	Knave	Knight	F	F	F	No: C would be a knight that lies

We can conclude that A is a knight, B is a spy, and C is a knave.