ECS20
Discussion 3: October 6-12, 2016

Exercise 1

Let p and q be two propositions. The proposition p *NOR* q is true when both p and q are false, and it is false otherwise. It is denoted $p \downarrow q$

a) Write down the truth table for $p \downarrow q$
b) Show that $p \downarrow q$ is logically equivalent to $\neg(p \vee q)$
c) Find a compound proposition logically equivalent to $p \rightarrow q$ using only the logical operator $\downarrow$

Exercise 2

Let $P(x)$ be the statement "$x=x^2$". If the domain consists of the integers, what are the truth values of the following statements:

a) P(0) b) P(1)
c) P(2) d) P(-1)
e) $\exists x \quad P(x)$ f) $\forall x \quad P(x)$

Exercise 3

Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase "It is not the case that.")

a) All dogs have fleas.
b) There is a horse that can add.
c) Every koala can climb.
d) No monkey can speak French.
e) There exists a pig that can swim and catch fish.

Exercise 4

a) Let a and b be two real numbers. Prove that if $n = ab$, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$
b) Prove or disprove that there is a rational number x and an irrational number y such that x^y is irrational.
c) Prove that $\sqrt[3]{2}$ is irrational
d) There exists no integer a and b such that $21a+30b = 1$