Discussion 4: Solutions

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Patrice Koehl
koehl@cs.ucdavis.edu

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Exercise 1

We use a proof by membership.

Let $x \in B \cap C$; by definition of the intersection, $x \in B$ and $x \in C$. Since $x \in B$ and $B \subseteq A$, we conclude that $x \in A$. Therefore $B \cap C \subseteq A$.

Let $x \in B \cup C$; by definition of the union, $x \in B$ or $x \in C$. If $x \in B$ and $B \subseteq A$, we conclude that $x \in A$. If $x \in C$ and $C \subseteq A$, we conclude that $x \in A$. In all cases, $x \in A$. Therefore $B \cup C \subseteq A$.

Exercise 2

- **a**. Let us define $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$. The $A - B = \{1, 2\}$ and $B - A = \{5, 6\}$.
- **b**. Let us define $A = \{1, 2, 3, 4\}$, $B = \{5, 6, 7\}$ and $C = \{3, 4, 8, 9\}$. Then $(A \cap B) \cup C = C = \{3, 4, 8, 9\}$ and $(A \cap C) \cup B = \{3, 4, 5, 6, 7\}$.

Exercise 3

We use a membership table:

A	B	$A - B$	$B - A$	$(A - B) \cup (B - A)$	$A \cup B$	$A \cap B$	$(A \cup B) - (A \cap B)$
1	1	0	0	0	1	1	0
1	0	1	0	1	1	0	1
0	1	0	1	1	1	0	1
0	0	0	0	0	0	0	0

Column 5 and 8 are equal: the two sets are equal.

Exercise 4

The only difficulty is to translate the problem in math.

We define D the set of tiles (this is the "universe", or "domain"), S the subset of squares, T the set of triangles, R the set of red tiles and B the set of blue tiles.

We know:

- $|D| = 19$ (there are 19 tiles total)
- $|S| = 12$ (there are 12 squares)
- $|R| = 11$ (there are 11 red tiles)
- $|S \cap B| = 4$ (there are 4 blue squares)
- $S \cup T = D$ and $S \cap T = \emptyset$
- $R \cup B = D$ and $R \cap B = \emptyset$

We directly deduce:

- $|D| = |S| + |T|$, therefore $|T| = 7$
- $|D| = |R| + |B|$, therefore $|B| = 8$
- $S = (S \cap R) \cup (S \cap B)$ and $(S \cap R) \cap (S \cap B) = S \cap R \cap B = \emptyset$, therefore $|S \cap R| = |S| - |S \cap B| = 8$
- A) The number of tiles that are square or blue:

  First, we observe that there are 8 blue tiles. Then: $|S \cup B| = |S| + |B| - |S \cap B| = 12 + 8 - 4 = 16$
- B)The number of tiles that are triangles and red:

  $|T \cap R| = 3$
- C) The number of tiles that are red or squares: $|S \cup R| = |S| + |R| - |S \cap R| = 12 + 11 - 8 = 15$

Exercise 5

Show that $\overline{A - B} = \overline{A} \cup B$.

We use a membership table:

A	B	$A - B$	$\overline{A - B}$	$\overline{A}$	$\overline{A} \cup B$
1	1	0	1	0	1
1	0	1	0	0	0
0	1	0	1	1	1
0	0	0	1	1	1

Column 4 and 6 are equal: the two sets are equal.