

## ECS20

Discussion 5: 10/20 to 10/26 2016

### Exercise 1

Determine whether each of these functions is a bijection from  $\mathbb{R}$  to  $\mathbb{R}$ :

- a)  $f(x) = -3x+4$
- b)  $f(x) = x^2+1$
- c)  $f(x) = (x+1)/(x+2)$
- d)  $f(x) = (x^2+1)/(x^2+2)$

### Exercise 2

Let  $S = \{-1, 0, 2, 4, 7\}$ . Find  $f(S)$  if:

- a)  $f(x) = 1$
- b)  $f(x) = 2x+1$
- c)  $f(x) = \left\lfloor \frac{x}{5} \right\rfloor$
- d)  $f(x) = \left\lfloor \frac{x^2+1}{3} \right\rfloor$

### Exercise 3

Let  $S$  be a subset of a universe  $U$ . The characteristic function  $f_S$  of  $S$  is the function from  $U$  to the set  $\{0,1\}$  such that  $f_S(x)=1$  if  $x$  belongs to  $S$  and  $f_S(x) = 0$  if  $x$  does not belong to  $S$ . Let  $A$  and  $B$  be two sets. Show that for all  $x$  in  $U$ ,

- a)  $f_{A \cap B}(x) = f_A(x) \cdot f_B(x)$
- b)  $f_{A \cup B}(x) = f_A(x) + f_B(x) - f_A(x) \cdot f_B(x)$

### Exercise 4

Let  $n$  be an integer. Show that  $\left\lfloor \frac{n}{2} \right\rfloor \left\lfloor \frac{n}{2} \right\rfloor = \left\lfloor \frac{n^2}{4} \right\rfloor$