Exercise 1

Let *a, b*, and *c* be three integers, with *a* non zero. Show that if $a \mid bc$ and $gcd(a,b) = 1$, then $a \mid c$.

Exercise 2

Let *n* be a natural number. We call *s(n)* the sum of its digits. Show that if *s(n)=s(3n)*, then *9 / n*. (*Hint:* a number *n* is divisible by 3 if and only if *s(n)* is divisible by 3. Similarly, a number *n* is divisible by 9 if and only if *s(n)* is divisible by 9).

Exercise 3

Let *a* be a non-zero integer. Show that if 2 does not divide *a* and 3 does not divide *a*, then $24 \mid (a^2 + 23)$.

Exercise 4

Prove that for every three natural numbers *x, y* and *z* strictly greater than 1, there is some natural number larger than *x, y* and *z* that is not divisible by *x, y* or *z*.