

# Discussion 9

ECS 20 (Fall 2016)

Patrice Koehl  
koehl@cs.ucdavis.edu

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## Exercise 1:

How many strings are there of five lowercase letters that have the letter  $x$  in them at least once?  
Let  $S_x$  be the set of such strings. Let  $S$  be the set of all strings of five lowercase letters, and let  $S_{nox}$  be the set of all string of five lowercase letters that do not contain the letter  $x$ .

Then:  $|S| = 5^{26}$

$|S_{nox}| = 5^{25}$

and finally,

$|S_x| = |S| - |S_{nox}| = 5^{26} - 5^{25}$

## Exercise 2:

a) How many strings of six lowercase letters from the English alphabet contain:

1) the letter  $a$ .

$$|S_a| = |S| - |S_{noa}| = 6^{26} - 6^{25}$$

2) the letter  $a$ .

We note first that we have the relation:

$$|S_{ab}| = |S| - |S_{noaornob}|$$

Let us compute now the number of strings that do not contain the letter  $a$  or do not contain the letter  $b$ . Using the extended sum rule,

$$|S_{noaornob}| = |S_{noa}| + |S_{nob}| - |S_{noaandnob}| = 6^{25} + 6^{25} - 6^{24}$$

Finally:

$$|S_{ab}| = 6^{26} - 6^{25} - 6^{25} + 6^{24}$$

b) How many bit strings contain exactly eight 0s and 10 1s if every 0 must be immediately followed by a 1.

Since each of the 0 need to be followed by a 1, the bit string will contain 8 pairs 01, and 2 additional 1s. Therefore we built a string with 10 positions, with two of the positions being 1s, the others being 01. There are 45 ways to build such strings.

c) How many ways are there to seat six people around a circular table?

There are  $6 \times 5 \times 4 \times 3 \times 2 \times 1$  ways to put the people on a line... since this line is circular, we need to divide this by 6... therefore there are  $5! = 120$  ways to seat six people around a circular table.

### Exercise 3:

A combination lock has three numbers in the combination, each in the range 1 to 40

- a) How many different combinations are there?

There are 40 possible values for each of the three numbers; using the product rules, the total number of combinations is:

$$T = 40 \times 40 \times 40 = 64,000$$

- b) How many of the combinations have no duplicate numbers? There are 40 possible values for the first number, 39 for the second number, and 38 for the last number as we do not want duplicates. Therefore, the total number of combinations with no duplicates is:

$$D = 40 \times 39 \times 38 = 59280$$

- c) How many of the combinations have exactly two of the three numbers matching?

The combination is defined by two numbers: the one that is duplicated, and a second number that is not equal to the first one. In addition, there are three possibilities for positioning the number that is not duplicated. Therefore, the total number of combinations that has exactly two of the three numbers matching is:

$$D2 = 3 \times 40 \times 39 = 4680.$$

*An alternate solution:*

There are three types of combinations: those without any duplicate (D), those with exactly two of the three numbers matching (D2), and those that have all three numbers matching (D3). It is easy to see that  $D3 = 40$ . Therefore:

$$T = D + D2 + D3$$

$$D2 = T - D - D3 = 64000 - 59280 - 40 = 4680.$$