# Review session

ECS 20 (Fall 2016)

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## 1 Simple propositions

For each proposition on the left, indicate if it is a tautology or not:

Table 1: Propositional logic

| Proposition | Tautology (Yes/ No) |
|---|---|
| $(\neg(p \wedge q)) \leftrightarrow (\neg p \vee \neg q)$ | |
| $(\neg(p \wedge q)) \leftrightarrow (\neg p \wedge \neg q)$ | |
| $(\neg(p \vee q)) \leftrightarrow (\neg p \wedge \neg q)$ | |
| if $6^2 = 36$ then $2 = 3$ | |
| if $6^2 = 36$ then $gcd(10, 5) = 5$ | |
| if $6^2 = -1$ then $36 = -1$ | |

## 2 Knights and Knaves

A very special island is inhabited only by Knights and Knaves. Knights always tell the truth, while Knaves always lie. You meet three inhabitants: Alex, John and Sally. Alex says, "If John is a Knight then Sally is a Knight". John says, "Alex is a Knight and Sally is a Knave". Can you find what Alex, John, and Sally are? Explain your answer.

## 3 Proofs: direct, indirect, and contradictions

### 3.1 Different methods of proofs

Let $n$ be an integer. Show that if $3n^2 + 2n + 9$ is odd, then $n$ is even using a direct, indirect, and proof by contradiction.

### 3.2 Proof by contradiction

Let $n$ be a strictly positive integer. Show that $\frac{6n+1}{2n+4}$ is not an integer

### 3.3 Proof by contradiction

Let $n$ be a strictly positive integer. Show that if $\sqrt{n^2+1}$ is not an integer.

### 3.4 Number theory: gcd

Let $a$ and $b$ be two strictly positive integers. Show that if $\gcd(a,b)=1$ then $\gcd(a,b^2)=1$.

### 3.5 Number theory: gcd

Let $a$ and $b$ be two strictly positive integers. Show that if $\gcd(a,b)=1$ then $\gcd(a^2,b^2)=1$ using a proof by contradiction.

### 3.6 Number theory: divisibility

Let $a$ and $b$ be two strictly positive integers with $\gcd(a,b)=1$. Let $c$ be another strictly positive integers. Show that if $a/c$ and $b/c$, then $ab/c$.

## 4 Functions

### 4.1 Function floor

Let $n$ be an integer and $x$ a real number. Show that $n \leq x$ if and only if $n \leq \lfloor x \rfloor$.

### 4.2 Function floor

Let $x$ be a real number. Show that the integer $\lfloor x+\frac{1}{2}\rfloor + \lfloor x-\frac{1}{2}\rfloor$ is odd.

### 4.3 Function floor

Find the remainder of the division of $90^{1000}$ by 11.

## 5 Proofs by induction

### 5.1 Identity

a) Show that $1+3+\ldots 2n-1=n^2$, for all $n \geq 1$.

b) Show that $\displaystyle\sum_{k=1}^{n} \frac{1}{4k^2-1} = \frac{n}{2n+1}$ for all integer $n \geq 1$.

### 5.2 Divisibility

a) Show that $5/(7^n-2^n)$ for all integer $n \geq 1$.
b) Show that $6/[n(2n+1)(7n+1)]$ for all integer $n \geq 1$.

### 5.3 Stamps

Use induction to prove that any postage of $n$ cents (with $n \geq 18$) can be formed using only 3–cent and 10–cent stamps.

### 5.4 Other

Prove by induction that for all $n \geq 1$, there exist two strictly positive integers $a_n$ and $b_n$ such that $(1+\sqrt{2})^n = a_n + b_n\sqrt{2}$.

### 5.5 Fibonacci

Let $f_n$ be the Fibonacci numbers. show that $f_{n-1}f_{n+1} - f_n^2 = (-1)^n$, for all $n > 1$.

## 6 Counting

### 6.1 Bitstrings

a) How many bit strings of length $n$ can we form that contain at least one 0 and one 1? b) How many bit strings of length 4 do not contain three consecutive 0s?

### 6.2 Anagrams

a) How many words can you form with the letters of the word PATRICE? b) How many words can you form with the letters of the word PATRICE that start with a consonant and end with a vowel?

### 6.3 Subsets

Let $E$ be a set with $n$ elements and let $A$ be a subset of $E$ with $p$ elements. How many subsets of $E$ contain three, and only three elements of $A$ (among possibly other elements of $E$)? (Reminder: there are $\dfrac{p(p-1)(p-2)}{6}$ subsets of 3 elements in a set with $p$ elements.)

### 6.4 Words

Let $A = \{\alpha, \beta, \gamma\}$ be a set with three elements. We call $\alpha$, $\beta$, and $\gamma$ "letters". How many words of length 4 can we form with only letters from $A$ that contain at least one of each letter from $A$?