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ECS 20: Discrete Mathematics
Finals
Dec 9 2016

Notes:

1) Finals are open book, open notes.
2) You have two hours, no more: I will strictly enforce this.
3) You can answer directly on these sheets (preferred), or on loose paper.
4) Please write your name at the top right of each page you turn in!
5) Please, check your work!
6) There are 4 parts with a total possible number of points of 110, with an additional 5 points extra credit.

Part I: logic (2 questions, each 10 points; total 20 points)

1) For each of the five propositions in the table below, indicates on the right if they are **tautologies or not** (p and q are propositions).

Proposition	**Tautology (Yes or No)**
If 1+3 = 5 then 3 = 6	
$(p \wedge \neg p) \to q$	
$(p \vee \neg p) \to (q \wedge \neg q)$	
if 2+2 = 4 then 4=5	
$(p \wedge \neg q) \vee (\neg p \vee q)$	

3) Let us play a logical game. You find yourself in front of two rooms whose doors are closed. If a lady is in Room I, then the sign on the door is true, but if a tiger is in it, the sign is false. In Room II, the situation is the opposite: a lady in the room means the sign on the door is false, and a tiger in the room means the sign is true. It is possible that both rooms contain ladies or both rooms contain tigers, or that one room contains a lady and the other a tiger. Here are the signs:

I	II
Both rooms contain Ladies	Both rooms contain Ladies

Can you say what each room contains? Justify your answer

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Part II: proofs and number theory (4 questions; each 10 points; total 40 points)

1) Let a and b be **two strictly positive real numbers**. Use a proof by contradiction to show that if $\frac{a}{1+b} = \frac{b}{1+a}$ then $a = b$.

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2) Let *n* be a **natural number**. Show that *n(n+1)* is divisible by 2.

3) Let *A, B*, and *C* be three sets in a domain *D*. Use a proof by contradiction to show that if $A \subset B$ and $B \cap C = \varnothing$ then $A \cap C = \varnothing$.

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4) Evaluate the remainder of the division of 14^{3139} by 17.

Part III. Proof by induction (3 questions; each 10 points; total 30 points)

1) Prove by induction that $3^{2n+1} + 2^{n+2}$ is divisible by 7, for all integers $n \geq 0$

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2 Prove by induction that $4^n - 1$ is divisible by 5, whenever **n** is a **strictly positive even integer**.

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3) Let x and y be two strictly positive integers. We consider the equation:

$$(x^2 + xy - y^2)^2 = 1 \qquad (1)$$

if the pair (x,y) satisfies this Equation (1), it is called a solution to (1).

Let now a_n be the sequence defined by: $a_0 = a_1 = 1$ and $a_{n+2} = a_{n+1} + a_n$. Show by induction that (a_n, a_{n+1}) is a solution to Equation (1) for all $n \geq 0$.

Part IV. Counting. (2 problems; each 10 points; total 20 points)

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1) Let $A = \{a,b,c\}$ be a set with three elements. We call *a, b,* and *c* "letters". How many words of length *n* can we form with only letters from A that contain at least one of each letter from A?

2) Show that if **eleven distinct integer numbers** are selected in the set $S=\{1,2,\ldots,20\}$, there are (at least) two of them whose difference is equal to 5.

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Part V. Extra credit (one problem, 5 points)

Let n be a strictly positive integer. Use **strong induction** to show that $\sum_{i=1}^{n}(-1)^i i^2 = (-1)^n \sum_{i=1}^{n} i$ for all $n \geq 0$