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ECS 20: Discrete Mathematics
Finals
March 21, 2006

Notes:

1) Finals are open book, open notes. No computers though…
2) You have two hours, no more: I will strictly enforce this.
3) You can answer directly on these sheets (preferred), or on loose paper.
4) Please write your name at the top right of each page you turn in!
5) Please, check your work!
6) There are 4 parts with a total possible number of points of 70. I will grade however over a total of 65, i.e. one question can be considered "extra credit". You choose!

Part I: logic (2 questions, first 6 points, second 4 points; total 10 points)

1) Show in **two** different ways that the propositions $((\neg p) \wedge (\neg p \rightarrow q))$ and $(\neg p \wedge q)$ are equivalent.

2) What is wrong with this sentence: "Public transportation is necessary because everyone needs it"?

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Part II: proofs and number theory (4 questions; each 5 points; total 20 points)

1) Prove or disprove that 2^n+1 is prime for all non negative integer n

2) Show that $\sqrt[3]{3}$ is irrational

3) Show that if 9 divides 10^{n-1}-1, then 9 divides 10^n-1

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4) Show that $n^2 - n + 5$ is odd for all integer n.

Part III. Proof by induction (4 questions; each 5 points; total 20 points)

1) Show by induction that $\sum_{i=1}^{n} \left(i\ 2^i \right) = (n-1)\ 2^{n+1} + 2$ for all n≥1

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2) Guess a closed formula, in terms of n, for the sequence given by $a_1=1$, and for $k\geq 2$, $a_k = a_{k-1} + (2k-1)$. Use induction to prove your guess is correct. (Hint: write at least the first 5 terms of the sequence).

3) Prove by induction that $3^{4m} \equiv 1 \pmod{10}$ for all $m\geq 1$

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4) What is wrong with the following proof:
We want to show by induction that $\forall n > 0 \quad P(n)$ is true, where P(n) is defined as:

$$P(n): \quad \sum_{i=1}^{n} i = \frac{(2n+1)^2}{8}$$

Basis step: P(1) is true

Inductive step: We suppose that P(k) is true, for k>0, i.e. $\sum_{i=1}^{k} i = \frac{(2k+1)^2}{8}$

Then: $$\sum_{i=1}^{k+1} i = \left(\sum_{i=1}^{k} i\right) + (k+1) = \frac{(2k+1)^2}{8} + \frac{8k+8}{8} = \frac{4k^2+12k+9}{8} = \frac{(2(k+1)+1)^2}{8}$$

Hence P(k+1) is true.
According to the principle of mathematical induction, P(n) is true for all n>0.

Part IV. Counting. Pigeonhole principle. (3 problems; first and second 5 points, third 10 points; total 20 points)

1) Prove that there are two powers of 3 whose difference is divisible by 2006.

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2) Given n integers $a_1, a_2, \ldots, a_n$, not necessarily distinct, there exist integers k and l with $0 \le k < l \le n$ such that the sum $a_{k+1}+a_{k+2}+\ldots+a_l$ is a multiple of n. (Hint: consider the numbers $S_m = a_1 + \cdots a_m$ for all m such that $1 \le m \le n$).

3) Passwords on a given system have to be eight characters long, with each character being either a lower case letter, or a digit between 0 and 9.
   a. How many passwords can you form that contain at least one digit?
   b. How many passwords can you form that contain at least one letter, and at least one digit?