**ECS 20: Discrete Mathematics**

**Midterm**

**October 19, 2016**

***Notes:***

1. Midterm is open book, open notes. No computers though…
2. You have 40 minutes, no more: We will strictly enforce this.
3. You can answer directly on these sheets (preferred), or on loose paper.
4. Please write your name at the top right of at least the first page that you turn in!
5. Please, check your work!

**Part I: logic (3 questions, each 10 points; total 30 points)**

**1)** Using truth table or logical equivalence, indicate which (if any) of the propositions below are tautologies or contradictions

a) 

b) 

2) Let us play a logical game. You find yourself in front of three rooms whose doors are closed. One of these rooms contains a Lady, another a Tiger and the third room is empty. There is one sign on each door; you are told that the sign on the door of the room containing the Lady is true, the sign on the door of the room with the Tiger is false, and the sign on the door of the empty room could be either true or false. Here are the signs:

II

The tiger is in room I

III

This room is empty

I

Room III is empty

Which room contains the Lady, which room contains the Tiger, and which room is empty? Justify your answer

**Part II: proofs (3 questions; each 10 points; total 30 points)**

1. Let *n* be an integer. Show that *n* is even if and only if  is even.
2. Let *n* be an integer. Show that  is even
3. Let x be a real number. Show that if  then x < 1

**Part III: extra credit (5 points)**

Prove that every rational number can be written as the product of two irrational numbers.