

Name: _____
ID: _____

ECS 20: Discrete Mathematics
Midterm
October 19, 2016

Notes:

- 1) Midterm is open book, open notes. No computers though...
- 2) You have 40 minutes, no more: We will strictly enforce this.
- 3) You can answer directly on these sheets (preferred), or on loose paper.
- 4) Please write your name at the top right of at least the first page that you turn in!
- 5) Please, check your work!

Part I: logic (3 questions, each 10 points; total 30 points)

1) Using truth table or logical equivalence, indicate which (if any) of the propositions below are tautologies or contradictions

a) $\neg(p \rightarrow \neg r) \vee [(\neg q) \leftrightarrow r]$

Name: _____

ID: _____

b) $[\neg(p \rightarrow \neg q)] \wedge [(\neg r \vee q) \leftrightarrow \neg p]$

2) Let us play a logical game. You find yourself in front of three rooms whose doors are closed. One of these rooms contains a Lady, another a Tiger and the third room is empty. There is one sign on each door; you are told that the sign on the door of the room containing the Lady is true, the sign on the door of the room with the Tiger is false, and the sign on the door of the empty room could be either true or false. Here are the signs:

I Room III is empty	II The tiger is in room I	III This room is empty
--	--	---

Which room contains the Lady, which room contains the Tiger, and which room is empty?
Justify your answer

Name: _____
ID: _____

Part II: proofs (3 questions; each 10 points; total 30 points)

- 1) Let n be an integer. Show that n is even if and only if $n + n^2 - n^3$ is even.

Name: _____

ID: _____

2) Let n be an integer. Show that $n^2 - n$ is even

3) Let x be a real number. Show that if $x^3 + x^2 - 2x < 0$ then $x < 1$

Name: _____
ID: _____

Part III: extra credit (5 points)

Prove that every rational number can be written as the product of two irrational numbers.