*Name:*___________________________________

*ID:*______ ____________________________

ECS 20: Discrete Mathematics
Midterm
April 20, 2016

Notes:

1) Midterm is open book, open notes. No computers though…
2) You have 50 minutes, no more: We will strictly enforce this.
3) You can answer directly on these sheets (preferred), or on loose paper.
4) Please write your name at the top right of at least the first page that you turn in!
5) Please, check your work!

Part I: logic (4 questions, each 10 points; total 40 points)

1) Using truth table or logical equivalence, indicate which (if any) of the propositions below are tautologies or contradictions

a) $\left[p \wedge (q \wedge r)\right] \rightarrow \left[\left((r \wedge p) \wedge q\right) \vee q\right]$

Name:___________________________________

ID:______ ____________________________

b) $\left[p \leftrightarrow (\neg q \wedge \neg r)\right] \rightarrow \left[(\neg(q \wedge r)) \rightarrow p\right]$

c) $\left[p \leftrightarrow (\neg q \wedge \neg r)\right] \rightarrow \left[(\neg(q \vee r)) \rightarrow p\right]$

Name:________________________________
ID:______ __________________________

2) *Smullyan's island.*

A very special island is inhabited only by Knights and Knaves. Knights always tell the truth, while Knaves always lie. You meet three inhabitants: Alex, John and Sally. Alex says, \`John is a Knight, if and only if Sally is a Knave'. John says, \`If Sally is a Knight, then Alex is a Knight'. Can you find what Alex, John, and Sally are? Explain your answer.

Part II: proofs (4 questions; each 10 points; total 40 points)

1) Use a constructive proof to show that there exist two distinct strictly positive integers whose sum and difference are both perfect squares (A perfect square is a number that can be expressed as the *product of two equal integers*; 4 is a perfect square as 4 = 2*2; 1 is a perfect square as 1 = 1*1).

*Name:*___________________________________

*ID:*______ ____________________________

2) Let a and b be two integers. Show that if either ab or $a+b$ is odd, then either a or b is odd.

3) Let a and b be two integers. Show that if $a^2(b^2-2b)$ is odd, then a is odd and b is odd

Name:____________________________________
ID:______ ____________________________

4) Use a proof by contradiction to show that if $(a,b) \in Z^2$ (i.e. a and b are integers), then $a^2 - 4b \neq 2$.

Part III: extra credit (5 points)

A very special island is inhabited only by Knights, Knaves, and Normals. Knights always tell the truth, knaves always lie, while Normals may tell the truth or lie. You meet three inhabitants: Alex, Bill, and Corinne. You know that at least one of them is a Knight, and at least one of them is a Knave. Alex says, `Bill is a Knave if and only if Corinne is a Knave.' Bill says, `If Alex is a Knave, then Corinne is a Knave.' Corinne claims, `Alex is a Knave or Bill is a Knight.' Can you find out what type of inhabitant Alex, Bill, and Corinne are? Explain your work.