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ECS 20: Discrete Mathematics
Midterm
November 14, 2016

Notes:

1) Midterm is open book, open notes. No computers though…
2) You have 40 minutes, no more: We will strictly enforce this.
3) You can answer directly on these sheets (preferred), or on loose paper.
4) Please write your name at the top right of at least the first page that you turn in!
5) Please, check your work!

Part I: sets (2 questions, each 10 points; total 20 points)

1) Using truth table or logical equivalence, indicate which (if any) of the propositions below are tautologies or contradictions

Let *A, B*, and *C* be three sets in a domain *D*. Show that $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$

*Name:*_____
*ID:*_____

2) Let A and B be two sets in a domain D. Show that $\overline{\left(A \cap \overline{B}\right) \cup \left(B \cap \overline{A}\right)} = \left(\overline{A} \cap \overline{B}\right) \cup \left(B \cap A\right)$

Part II: functions (2 questions; each 10 points; total 20 points)

1) Let x be a ***real number***. Solve $\lfloor 3x - 2 \rfloor = x$.

*Name:*_____

*ID:*_____

2) Let x be a ***real number***. Show that $\left\lfloor \frac{x}{2} \right\rfloor + \left\lfloor \frac{x+1}{2} \right\rfloor = \lfloor x \rfloor$

Part III: Number theory (2 questions; each 10 points; total 20 points)

1) Let a, b, and c be three natural numbers. Show that if b /a and c/a and gcd(b,c) = 1 then bc / a.

*Name:*_____
*ID:*_____ _____

2) Show that *13* divides $3^{126} + 5^{126}$.

*Name:*_____
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Part IV: extra credit (5 points)

Let x be a real number. Solve $x\lfloor x \rfloor = x^2 - \lfloor x \rfloor^2$ where $\lfloor x \rfloor$ is the floor function.