*Name:*___________________________________
*ID:*______ _____________________________

ECS 20: Discrete Mathematics
Midterm
February 15, 2006

Notes:

1) Midterm is open book, open notes. No computers though…
2) You have one hour, no more: I will strictly enforce this.
3) You can answer directly on these sheets (preferred), or on loose paper.
4) Please write your name at the top right of each page you turn in!
5) Please, check your work!
6) There are 3 parts with a total possible number of points of 66. I will grade however over a total of 60, i.e. one question can be considered "extra credit". You choose!

Part I: logic (2 questions, each 6 points; total 12 points)

1) Construct the truth table for each of the statement below, and indicate which (if any) are tautologies or contradictions

a) $(p \to \neg p) \to \neg p$

b) $(p \wedge \neg p) \leftrightarrow (q \wedge \neg q)$

c) $(p \vee \neg p) \leftrightarrow (q \vee \neg q)$

*Name:*___________________________________
*ID:*______ ____________________________

2) Using logical equivalences (preferred) or truth table, show that the statements below are tautologies:

a) $(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \vee q) \vee (\neg p \vee -q)$

b) $(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge q) \vee (\neg p \wedge -q)$

Part II: proofs and number theory (5 questions; question 1-4 each 6 points; question 5 and 6 8 points each; total 42 points)

1) Prove that n is even if and only if $5n^2+2$ is even, where n is a natural number.

Name:___________________________________
ID:______ ____________________________

2) Prove that for every three natural numbers x,y and z strictly greater than 1, there is some natural number larger than x, y and z that is not divisible by x, y or z.

3) Prove that if n^2 is not divisible by 4, then n is odd.

4) Let $a = 2^{1001}-5^{701}+7^{256}$, $b = 2^{1001}-5^{701}+7^{256}-1$ and $c = 2^{1001}-5^{701}+7^{256}+1$. Show that one of these three numbers is divisible by 3, and another one is divisible by 2. Is your proof constructive?

Name:___________________________________

ID:______ ______________________________

5) Let n be a natural number, that we write as $n=a_p 10^p+a_{p-1}10^{p-1}+\ldots+a_1 10+a_0$, where $a_p, \ldots a_0$ are the digits of n
   a. Show that if a_0 is divisible by 2, n is divisible by 2.

   b. Show that if $10a_1+a_0$ is divisible by 4, n is divisible by 4

   c. Show that if a_0 is divisible by 5, n is divisible by 5.

6) a) Show that any number n such that n≡ 3 (mod 4) must have one prime factor q with q≡3 (mod 4)
   (Hint: First notice that for any prime number p, either p≡1 (mod 4), or p≡3 (mod 4). Then write n as a product of prime numbers, and suppose that all these prime factors are NOT congruent to 3 modulo 4).

b) Deduce that there are infinitely many primes congruent to 3 modulo 4. (Hint: Suppose there is only a finite set $\{p_1, p_2, \ldots p_N\}$ of prime numbers that are congruent to 3 modulo 4, and consider $M = 4p_1p_2\ldots p_N - 1$. Apply 6a) on *M*).

Part III: Algorithms (1 question, 12 points)

Devise an algorithm that replace each term of a finite sequence of integers with the sum of the square of the terms preceding it in the sequence, leaving the first term identical.
Example: replace S={1,3,5,6,7} with {1,1, 10, 35, 71} (i.e. the first 1 stays identical; 3 is replaced with 1^2; 5 is replaced with 1^2+3^2; 6 is replaced with $1^2+3^2+5^2$ and 7 is replaced with $1^2+3^2+5^2+6^2$.
What is the complexity of your algorithm?