ECS20
Homework 1
Due September 28, 2016

Exercise 1

A ball and a bat cost $11.10 (total). The bat costs $10.0 more than the ball. How much does the ball cost?

Exercise 2
Hints:
- An integer number N is odd if it can be written in the form $N = 2q + 1$, where q is an integer number
- An integer number N is even if it can be written in the form $N = 2q$, where q is an integer number
- An integer number N is a multiple of an integer number k if there exists an integer number q such that $N = kq$
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Prove the following statements:
a) The sum of any three consecutive odd numbers is always a multiple of 3
b) The sum of any five consecutive even numbers is always a multiple of 10
c) Prove that if you add the squares of three consecutive integer numbers and then subtract two, you always get a multiple of 3.

Exercise 3

Roger is an amateur magician. In one of his tricks he invites people in the audience to think of a number (integer). He then asks them to carry out the following simple instructions:

   triple your number
then   *add 5*
then   *multiply the number you now have by itself*
then   *subtract 25*
then   *divide by 3*
then   *divide by your original number*

Based on the final number obtained, Roger can then "guess" the initial number.

Show that there is no magic in this. Justify your answer.

Exercise 4

Prove the following identities, where p, q, x, m, and n are real numbers:
a) $8(p-q)+3(p+q)=2(p+2q)+9(p-q)$
b) $x(m+n)+y(n-m)=m(x-y)+n(x+y)$
c) $(x+2)(x+10)-(x-5)(x-4)=21x$

d) $m^4-1=(m^2+1)(m^2-1)$

Extra credit (Google problem):

Four persons need to cross a bridge to get back to their camp at night. Unfortunately, they only have one flashlight and it only has enough light left for seventeen minutes. The bridge is too dangerous to cross without a flashlight, and it is strong enough to support only two persons at any given time. The flashlight cannot be thrown from one side of the bridge to the other. Each of the campers walks at a different speed. One can cross the bridge in 1 minute, another in 2 minutes, the third in 5 minutes, and the last one takes 10 minutes to cross.

Tell these people how they can make it across in 17 minutes.