Homework 2: Solutions

ECS 20 (Fall 2016)

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September 29, 2016

Exercise 1

Construct a truth table for each of these compound propositions:

a) $(p \wedge q) \rightarrow (p \vee q)$

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

b) $(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$

p	$\neg p$	q	$q \rightarrow \neg p$	$p \leftrightarrow q$	$(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$
T	F	T	F	T	F
T	F	F	T	F	F
F	T	T	T	F	F
F	T	F	T	T	T

c) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$\neg q$	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	T

Exercise 2

Construct a truth table for each of these compound propositions:

a) $(p \oplus q) \vee (p \oplus \neg q)$

p	q	$\neg q$	$p \oplus q$	$(p \oplus \neg q)$	$(p \oplus q) \vee (p \oplus \neg q)$
T	T	F	F	T	T
T	F	T	T	F	T
F	T	F	T	F	T
F	F	T	F	T	T

b) $(p \oplus q) \wedge (p \oplus \neg q)$

p	q	$\neg q$	$p \oplus q$	$(p \oplus \neg q)$	$(p \oplus q) \wedge (p \oplus \neg q)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	F

Exercise 3

A contestant in a TV game show is presented with three boxes, A, B, and C. He is told that one of the boxes contains a prize, while the two others are empty. Each box has a statement written on it:

Box A: The prize is in this box

Box B: The prize is not in box A

Box C: The prize is not in this box

The host of the show tells the contestant that only one of the statements is true. Can the contestant find logically which box contains the prize? Justify your answer.

The simplest approach to solve this problem is to check systematically if A, B, or C contains the prize. In each case, we test the validities of the three statements.

Box with prize	Box A prize is in this box	Box B prize is not in box A	Box C prize is not in this box
Box A	True	False	True
Box B	False	True	True
Box C	False	True	False

If the prize were in box A or B, two of the propositions would be true, while if the prize is in box C, only one proposition would be true. The latter is therefore true, and the prize is in box C.

Exercise 4

This exercise relate to the inhabitants of the island of knights and knaves created by Smullyan, where knights always tell the truth and knaves always lie. You encounter two people, A and B. Determine, if possible, what A and B are if they address you in the way described. If you cannot determine what these two people are, can you draw any conclusions?

A says The two of us are both knights, and B says A is a knave.

We proceed as in class. We check all possible "values" for A and B, as well as the veracity of their statements:

Line number	A	B	A says "The two of us are both knights"	B says "A is a knave"
1	Knight	Knight	T	F
2	Knight	Knave	F	F
3	Knave	Knight	F	T
4	Knave	Knave	F	T

We can eliminate:

- Line 1, as B would be a knight but he lies
- Line 2, as A would be a knight but he lies
- Line 4 as B would be a knave but he says the true

Line 3 is valid, and it is the only one. Therefore, A is a knave and B is a knight.

Exercise 5

Use truth tables to verify the associative laws:

a) $(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$

   We need to show that the two propositions $(p \vee q) \vee r$ and $p \vee (q \vee r)$ have the same truth values:

b) $(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$

   We need to show that the two propositions $(p \wedge q) \wedge r$ and $p \wedge (q \wedge r)$ have the same truth values:

p	q	r	$p \vee q$	$q \vee r$	$(p \vee q) \vee r$	$p \vee (q \vee r)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

p	q	r	$p \wedge q$	$q \wedge r$	$(p \wedge q) \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	T	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

Exercise 6

Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are equivalent. We show that these two statements have the same truth values:

p	q	$p \wedge q$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$(p \wedge q) \vee (\neg p \wedge \neg q)$	$p \leftrightarrow q$
T	T	T	F	F	F	T	T
T	F	F	F	T	F	F	F
F	T	F	T	F	F	F	F
F	F	F	T	T	T	T	T

Exercise 7

Show that $(\neg(p \vee F) \wedge (\neg q \wedge T)) \vee p$ and $q \to p$ are equivalent.

We will use logical equivalence. Let us define $A = (\neg(p \vee F) \wedge (\neg q \wedge T)) \vee p$. We proceed by equivalence:

$$
\begin{array}{rcl|c}
A & \Leftrightarrow & (\neg(p \vee F) \wedge (\neg q \wedge T)) \vee p & \text{Problem definition} \\
& \Leftrightarrow & (\neg p \wedge \neg q) \vee p & \text{Absorption law} \\
& \Leftrightarrow & (\neg p \vee p) \wedge (\neg q \vee p) & \text{Distributivity law} \\
& \Leftrightarrow & T \wedge (p \vee \neg q) & \text{Complement law} \\
& \Leftrightarrow & p \vee \neg q & \text{Absorption law} \\
& \Leftrightarrow & q \to p & \text{Definition of implication}
\end{array}
$$

Exercise 8

Use two different methods to how that $(\neg(q \to p)) \vee (p \wedge q)$ and q are equivalent.

We will use a table of truth and logical equivalence:

a) Table of truth

We show that the two statements $(\neg(q \to p)) \vee (p \wedge q)$ and q have the same truth values:

p	q	$q \to p$	$\neg(q \to p)$	$p \wedge q$	$(\neg(q \to p)) \vee (p \wedge q)$
T	T	T	F	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	F	T	F	F	F

b) We use logical equivalence. Let us define $A = (\neg(q \to p)) \vee (p \wedge q)$. We proceed by equivalence:

A	$\Leftrightarrow$	$(\neg(q \rightarrow p)) \vee (p \wedge q)$	Problem definition
	$\Leftrightarrow$	$(\neg(\neq q \vee p) \vee (p \wedge q)$	property of implication
	$\Leftrightarrow$	$(q \wedge \neg p) \vee (p \wedge q)$	De Morgan's law and double negation
	$\Leftrightarrow$	$(q \wedge \neg p) \vee (q \wedge p)$	Commutative law
	$\Leftrightarrow$	$q \wedge (\neg p \vee p)$	Distributive law
	$\Leftrightarrow$	$q \wedge T$	Identity law
	$\Leftrightarrow$	q	Identity law

Extra Credit

We are back on the island of knights and knaves (see exercise above). John and Bill are residents. John: if Bill is a knave, then I am a knight Bill: we are different Who is who?

We proceed as for exercise 5: we check all possible "values" for John and Bill, as well as the veracity of their statements. Note that John's statement is an implication.

Line number	John	Bill	John says "If Bill is a knave, then I am a knight"	Bill says "We are different"
1	Knight	Knight	T	F
2	Knight	Knave	T	T
3	Knave	Knight	T	T
4	Knave	Knave	F	F

We can eliminate:

- Line 1, as Bill would be a knight but he lies
- Line 2, as Bill would be a knave but he tells the truth
- Line 3 as John would be a knave but he says the true

Line 4 is valid, and it is the only one. Therefore, both John and Bill are knaves.