

**ECS20**  
**Homework 2**  
**Due October 5, 2016**

**Exercise 1**

Construct a truth table for each of these compound propositions:

- a)  $(p \wedge q) \rightarrow (p \vee q)$
- b)  $(q \rightarrow \neg p) \Leftrightarrow (p \Leftrightarrow q)$
- c)  $(p \Leftrightarrow q) \oplus (p \Leftrightarrow \neg q)$

**Exercise 2**

Construct a truth table for each of these compound propositions:

- a)  $(p \oplus q) \vee (p \oplus \neg q)$
- b)  $(p \oplus q) \wedge (p \oplus \neg q)$

**Exercise 3**

A contestant in a TV game show is presented with three boxes, A, B, and C. He is told that one of the boxes contains a prize, while the two others are empty. Each box has a statement written on it:

Box A: The prize is in this box

Box B: The prize is not in box A

Box C: The prize is not in this box

The host of the show tells the contestant that only one of the statements is true. Can the contestant find logically which box contains the prize? Justify your answer.

**Exercise 4**

This exercise relate to the inhabitants of the island of knights and knaves created by Smullyan, where knights always tell the truth and knaves always lie. You encounter two people, A and B. Determine, if possible, what A and B are if they address you in the way described. If you cannot determine what these two people are, can you draw any conclusions?

A says “The two of us are both knights”, and B says “A is a knave”.

**Exercise 5**

Use truth tables to verify the associative laws:

- a)  $(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$
- b)  $(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$

**Exercise 6**

Show that  $p \Leftrightarrow q$  and  $(p \wedge q) \vee (\neg p \wedge \neg q)$  are equivalent.

**Exercise 7**

Use either a truth table, or logical equivalences, to show the equivalence:

$$(\neg(p \vee F) \wedge (\neg q \wedge T)) \vee p \Leftrightarrow (q \rightarrow p)$$

**Exercise 8**

Use two different methods to show the equivalence:

$$(\neg(q \rightarrow p)) \vee (p \wedge q) \Leftrightarrow q$$

**Extra Credit:**

We are back on the island of knights and knaves (see exercise 4 above). John and Bill are residents.

John: if Bill is a knave, then I am a knight

Bill: we are different

Who is who?