**ECS20**

**Fall 2016**

**Homework 3**

**Exercise 1**

Show that this implication is a tautology, by using a truth table:

 

**Exercise 2**

Show that  is a tautology

**Exercise 3**

a) Let *x* be a real number. Show that if *x2* is irrational, then *x* is irrational.

b) Based on question *a)*, can you say that “if *x* is irrational, it follows that *x2* is irrational.”?

**Exercise 4:**

Prove that a square of an integer ends with a 0, 1, 4, 5 6 or 9. (*Hint:* let *n = 10k+l*, where l = 0, 1, …,9)

**Exercise 5:**

Prove that if n is a positive integer, then n is even if and only if 5n+6 is even.

**Exercise 6:**

Prove that either 3x10450 + 15 or 3x10450 + 16 is not a perfect square. Is your proof constructive, or non-constructive?

**Exercise 7:**

Prove or disprove that if *a* and *b* are rational numbers, then *ab* is also rational.

**Exercise 8**:

Prove that at least one of the real numbers *a1, a2, …, an* is greater than or equal to the average of these numbers. What kind of proof did you use?

**Exercise 9:**

The proof below has been scrambled. Please put it back in the correct order.

**Claim**: For all *n ≥ 9*, if n is a perfect square, then *n-1* is not prime.

Since *(n-1)* is the product of 2 integers greater than 1, we know *(n-1)* is not prime (1)

Since *m ≥ 3*, it follows that *m-1 ≥ 2* and *m+1 ≥ 4* (2)

Let n be a perfect square such that *n ≥ 9*  (3)

This means that *n-1 = m2-1 = (m-1)(m+1)* (4)

There is an integer *m ≥ 3* such that *n=m2* (5)

**Exercise 10**

Prove that these four statements are equivalent: (i) *n2* is odd, (ii) *1-n* is even, (iii) *n3* is odd, (iv) *n2+1* is even.

**Extra credit:**

Use Exercise 8 to show that if the first 10 strictly positive integers are placed around a circle, in any order, then there exist three integers in consecutive locations around the circle that have a sum greater than or equal to 17.