ECS20
Fall 2016
Homework 3

Exercise 1

Show that this implication is a tautology, by using a truth table:

$$[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$$

Exercise 2

Show that $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$ is a tautology

Exercise 3

a) Let x be a real number. Show that if x^2 is irrational, then x is irrational.

b) Based on question *a)*, can you say that "if x is irrational, it follows that x^2 is irrational."?

Exercise 4:

Prove that a square of an integer ends with a 0, 1, 4, 5 6 or 9. (*Hint:* let $n = 10k+l$, where l = 0, 1, …,9)

Exercise 5:

Prove that if n is a positive integer, then n is even if and only if 5n+6 is even.

Exercise 6:

Prove that either $3 \times 10^{450} + 15$ or $3 \times 10^{450} + 16$ is not a perfect square. Is your proof constructive, or non-constructive?

Exercise 7:

Prove or disprove that if a and b are rational numbers, then a^b is also rational.

Exercise 8:
Prove that at least one of the real numbers $a_1, a_2, \ldots, a_n$ is greater than or equal to the average of these numbers. What kind of proof did you use?

Exercise 9:
The proof below has been scrambled. Please put it back in the correct order.

Claim: For all $n \geq 9$, if n is a perfect square, then $n-1$ is not prime.

Since $(n-1)$ is the product of 2 integers greater than 1, we know $(n-1)$ is not prime (1)

Since $m \geq 3$, it follows that $m-1 \geq 2$ and $m+1 \geq 4$ (2)

Let n be a perfect square such that $n \geq 9$ (3)

This means that $n-1 = m^2-1 = (m-1)(m+1)$ (4)

There is an integer $m \geq 3$ such that $n=m^2$ (5)

Exercise 10
Prove that these four statements are equivalent: (i) n^2 is odd, (ii) $1-n$ is even, (iii) n^3 is odd, (iv) n^2+1 is even.

Extra credit:
Use Exercise 8 to show that if the first 10 strictly positive integers are placed around a circle, in any order, then there exist three integers in consecutive locations around the circle that have a sum greater than or equal to 17.