ECS20
Homework 4

Proofs:

Exercise 1:
Give a direct proof, an indirect proof, and a proof by contradiction of the statement: « if n is even, then n+4 is even ».

Set Theory:

Exercise 2:
Let A, B and C be sets. Show that (A-B)-C = (A-C) - (B-C)

The symmetric difference of A and B, denoted by $A \oplus B$, is the set containing those elements in either A or B, but not in both A and B

Exercise 3:
Show that $A \oplus B = (A - B) \cup (B - A)$

Exercise 4
a) Show that $A \oplus B = B \oplus A$
b) Show that $(A \oplus B) \oplus B = A$
c) Show that $A \neq A \oplus A$ if A is a non empty set.

Exercise 5
Can you conclude that A = B if A, B, and C are sets such that:
a) $A \cup C = B \cup C$
b) $A \cap C = B \cap C$

The cardinality of a finite set A, denoted as |A|, is the number of elements it contains. There is a nice property on cardinality that we will suppose known:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Exercise 6
Show that if A, B, and C are sets then
$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

Exercise 7
Let A and B be subsets of the finite universal set U. Show that:
$$\left|\overline{A} \cap \overline{B}\right| = |U| - |A| - |B| + |A \cap B|$$

Exercise 8
Let A_i={…,-2,-1,0,1,…,i}. Find:

a) $\bigcup_{i=1}^{n} A_i$

b) $\bigcap_{i=1}^{n} A_i$

Exercise 9
Let A and B be two sets. Show that if $A \cup B = B$ then $A \cap B = A$

Exercise 10
Let A and B be two sets. Show that if $A \cap B = A$ then $B \cap \left(\overline{B \cap \overline{A}}\right) = A$

*****Extra credit:***

a) Let A and B be two sets. Show that $P(A) \cap P(B) = P(A \cap B)$, where P means the power set.
b) Give one example of two sets A and B such that $P(A) \cup P(B) \neq P(A \cup B)$