**ECS20**
**Homework 5**

## Exercise 1

Find these values :

*a*) $\lfloor 2.4 \rfloor$ *b*) $\lceil 2.4 \rceil$ *c*) $\lfloor -3.4 \rfloor$

*d*) $\lceil -3.4 \rceil$ *e*) $\lceil 6.99 \rceil$ *f*) $\lceil -6.99 \rceil$

*g*) $\left\lfloor \frac{1}{4} + \left\lceil \frac{1}{4} \right\rceil \right\rfloor$ *h*) $\left\lceil \left\lfloor \frac{1}{4} \right\rfloor + \left\lceil \frac{1}{4} \right\rceil + \frac{1}{2} \right\rceil$

## Exercise 2 (proof)

a) Show that the following statement is true:

"If x is a real number such that $x^2+2=0$, then $x^4 = -5$".

b) Constructive proof:

"If x and y are real numbers such that x < y, show that there exists a real number z with x <z < y"

## Exercise 3

Let *x* be a real number. Show that $\lfloor 3x \rfloor = \lfloor x \rfloor + \left\lfloor x + \frac{1}{3} \right\rfloor + \left\lfloor x + \frac{2}{3} \right\rfloor$

## Exercise 4

Show that for all strictly positive integer *n* and for all real number *x*, $\left\lfloor \frac{\lfloor nx \rfloor}{n} \right\rfloor = \lfloor x \rfloor$

*****Extra credit:***

Let us consider a generalization of exercise 3. Let x be a real number, and N an integer greater or equal to 3. Show that:

$$\lfloor Nx \rfloor = \lfloor x \rfloor + \left\lfloor x + \frac{1}{N} \right\rfloor + \left\lfloor x + \frac{2}{N} \right\rfloor + \ldots + \left\lfloor x + \frac{N-1}{N} \right\rfloor$$