ECS20
Homework 6

Exercise 1

Prove or disprove each of these statements about the floor and ceiling functions.
a) $\lceil \lfloor x \rfloor \rceil = \lfloor x \rfloor$ for all real numbers x.
b) $\lfloor xy \rfloor = \lfloor x \rfloor \lfloor y \rfloor$ for all real numbers x and y.
c) $\left\lfloor \sqrt{\lceil x \rceil} \right\rfloor = \left\lfloor \sqrt{x} \right\rfloor$ for all positive real numbers x.

Exercise 2

Show that x^3 is $O(x^4)$ but that x^4 is not $O(x^3)$.

Exercise 3

a) Show that $2x-9$ is $\Theta(x)$.
b) Show that $3x^2 + x - 5$ is $\Theta(x^2)$
c) Show that $\left\lfloor x + \frac{2}{3} \right\rfloor$ is $\Theta(x)$
d) Show that $log_{10}(x)$ is $\Theta(log_2(x))$

Exercise 4

Let a and b be two integers. Use a proof by contradiction to show that if a^2-b^2+2ab is odd then $a-b$ is odd.

Exercise 5

Use a proof by contradiction to show that:

There exists a strictly positive real number r such that for all real number x, if $x - \lfloor x \rfloor < r$ then $\lfloor 3x \rfloor = 3x$.

Extra credit:

We call a positive integer perfect if it equals the sum of its positive divisors other than itself.
a) Show that 6 and 28 are perfect
b) Show that $2^{p-1}(2^p-1)$ is a perfect number when 2^p-1 is prime.