**ECS20**

**Homework 7: Number theory**

**Exercise 1:**

What are the quotient and remainder when:

a) -2002 is divided by 89?

b) 0 is divided by 19?

c) 1,234,567 is divided by 101?

d) -100 is divided by 103?

**Exercise 2:**

a) Let a be a positive integer. Show that *gcd(a,a-1) = 1*.

b) Use the result of part a) to solve the Diophantine equation

*a+2b=2ab*

where (a,b) are positive integers.

**Exercise 3:**

Let *a*, *b*, and *c* be three integers. Show that the equation *ax + by = c* has at least one solution *(x1,y1)* if and only if *gcd(a,b) / c*.

**Exercise 4:**

Let a, b and n be three positive integers with *gcd(a,n) = 1* and *gcd(b,n) = 1*. Show that *gcd(ab,n) = 1*

**Exercise 5:**

Prove that there are no solutions in integers *x* and *y* to the equation *3x2+5y2=19*. (*Hint:* consider this equation modulo 5)

**Exercise 6:**

Show that if *n>3* then *n, 2n+1* and 4*n+1* cannot all be prime (*Hint:* consider the division of *n* by 3)

**Exercise 7:**

Prove or disprove that there are three consecutive odd positive integers that are primes, that is, odd primes of the form *p*, *p+2*, *p+4*.

**Exercise 8:**

Prove that if *n* is a positive integer such that the sum of its divisors is *n+1*, then *n* is prime.

**Extra credit:**

Let *a* and *b* be two strictly positive integers. Solve *gcd(a,b)+lcm(a,b) = b + 9*