ECS20 Homework 7: Number theory

Exercise 1:

What are the quotient and remainder when:
a) -2002 is divided by 89?
b) 0 is divided by 19?
c) 1,234,567 is divided by 101?
d) -100 is divided by 103?

Exercise 2:

a) Let a be a positive integer. Show that gcd(a,a-1) = 1.
b) Use the result of part a) to solve the Diophantine equation a+2b=2ab
where (a,b) are positive integers.

Exercise 3:

Let *a*, *b*, and *c* be three integers. Show that the equation ax + by = c has at least one solution (x_l, y_l) if and only if gcd(a,b) / c.

Exercise 4:

Let a, b and n be three positive integers with gcd(a,n) = 1 and gcd(b,n) = 1. Show that gcd(ab,n) = 1

Exercise 5:

Prove that there are no solutions in integers x and y to the equation $3x^2+5y^2=19$. (*Hint:* consider this equation modulo 5)

Exercise 6:

Show that if n > 3 then n, 2n+1 and 4n+1 cannot all be prime (*Hint:* consider the division of n by 3)

Exercise 7:

Prove or disprove that there are three consecutive odd positive integers that are primes, that is, odd primes of the form p, p+2, p+4.

Exercise 8:

Prove that if *n* is a positive integer such that the sum of its divisors is n+1, then *n* is prime.

Extra credit:

Let *a* and *b* be two strictly positive integers. Solve gcd(a,b)+lcm(a,b) = b + 9