ECS20
Homework 8: Induction

Proofs by induction.

Exercise 1:

Show that $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$ for all n ≥ 1.

Exercise 2:

Use mathematical induction to prove that:

$$1*2*3+2*3*4+\ldots+n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$$ for all n ≥ 1.

Exercise 3:
Prove that:

$$1+\frac{1}{4}+\frac{1}{9}+\ldots+\frac{1}{n^2}<2-\frac{1}{n}$$
Whenever n is a positive integer greater than 1.

Exercise 4:

Use mathematical induction to show that $n^2\text{-}7n\text{+}12$ is non negative if n is an integer greater than 3.

Exercise 5:
Use mathematical induction to prove that a set with n elements has $n(n-1)/2$ subsets containing exactly two elements whenever n is an integer greater than or equal to 2.

Exercise 6:

Find the flaw with the following proof that $a^n=1$ for all non negative integer n, whenever a is a non zero real number:
Basis step: $a^0=1$ is true, by definition of a^0
Inductive step: assume that $a^j=1$ for all non negative integers j with $j \leq k$. Then note that:

$$a^{k+1}=\frac{a^k a^k}{a^{k-1}}=\frac{1.1}{1}=1$$

Exercise 7:
Use mathematical induction to show that 21 divides $4^{n+1} + 5^{2n-1}$ whenever *n* is a positive integer.

Recurrence

In the following exercises, f_n is the nth Fibonacci number.

Exercise 8:
Prove that $f_1^2 + f_2^2 + \cdots f_n^2 = f_n f_{n+1}$ whenever n is a positive integer.

Exercise 9:
Show that $f_0 - f_1 + f_2 - \cdots - f_{2n-1} + f_{2n} = f_{2n-1} - 1$ whenever n is a positive integer.

Extra credit:
Use mathematical induction to prove that a set with *n* elements has *n(n-1)(n-2)/6* subsets containing exactly three elements whenever *n* is an integer greater than or equal to 3.