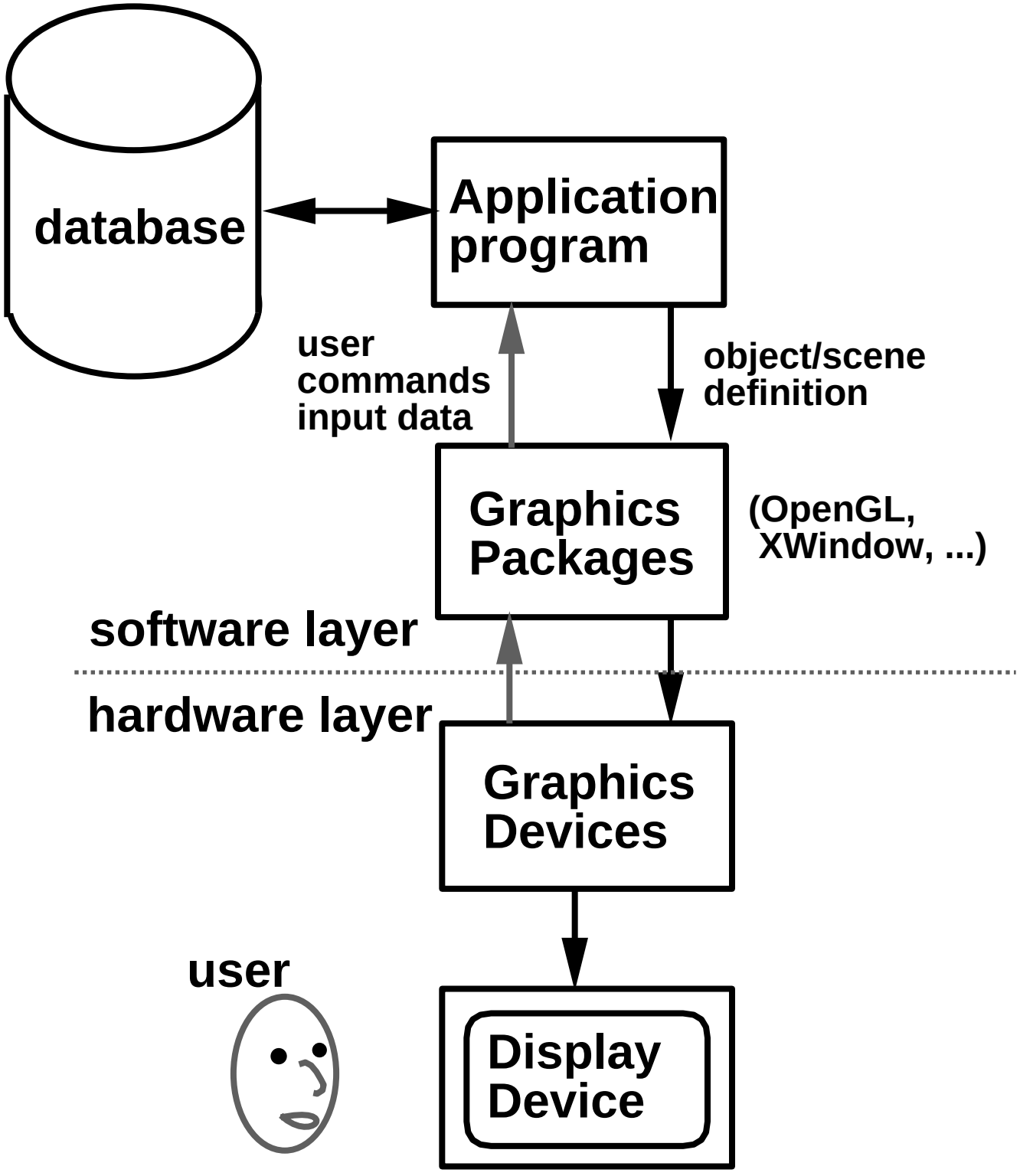
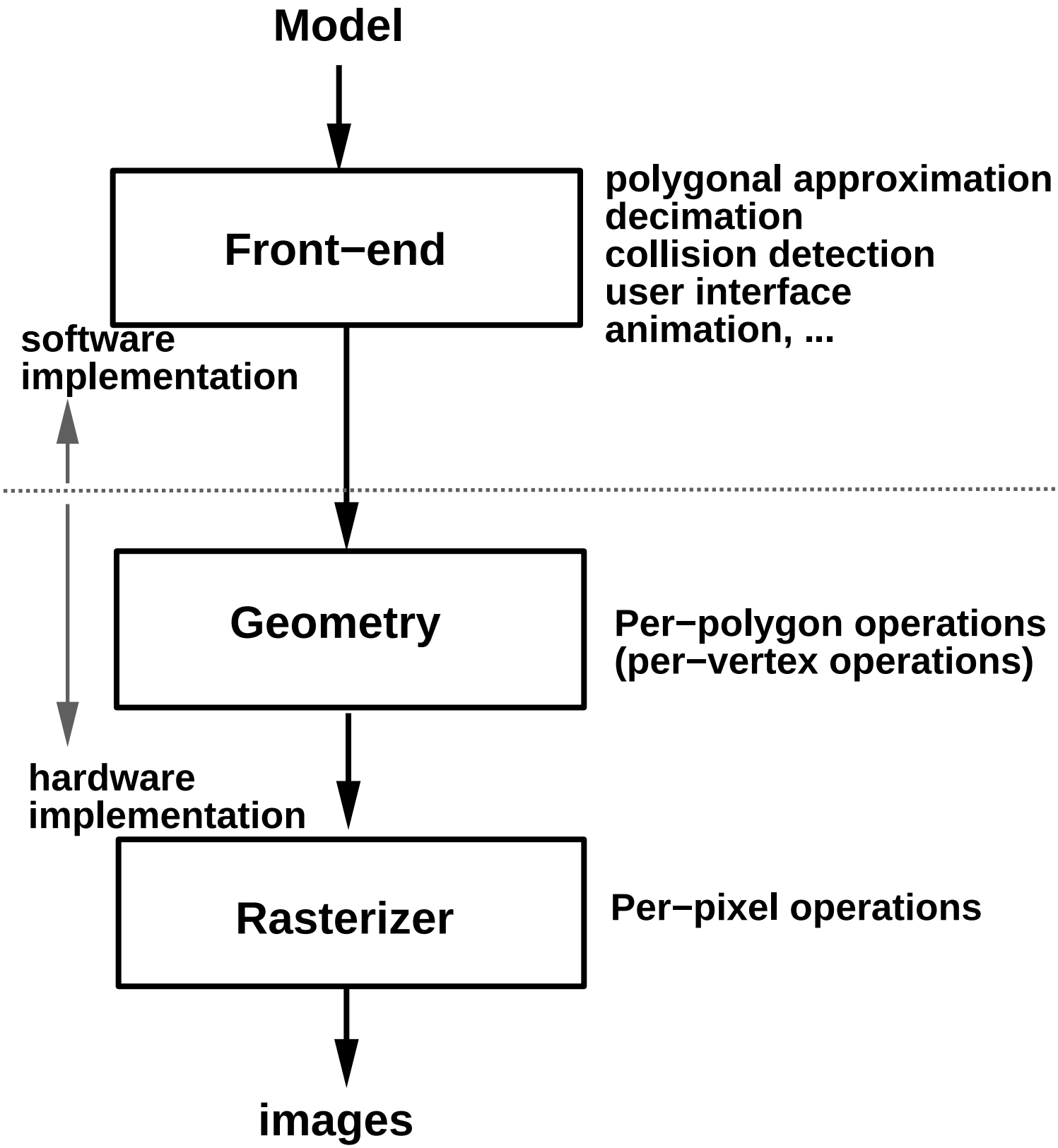
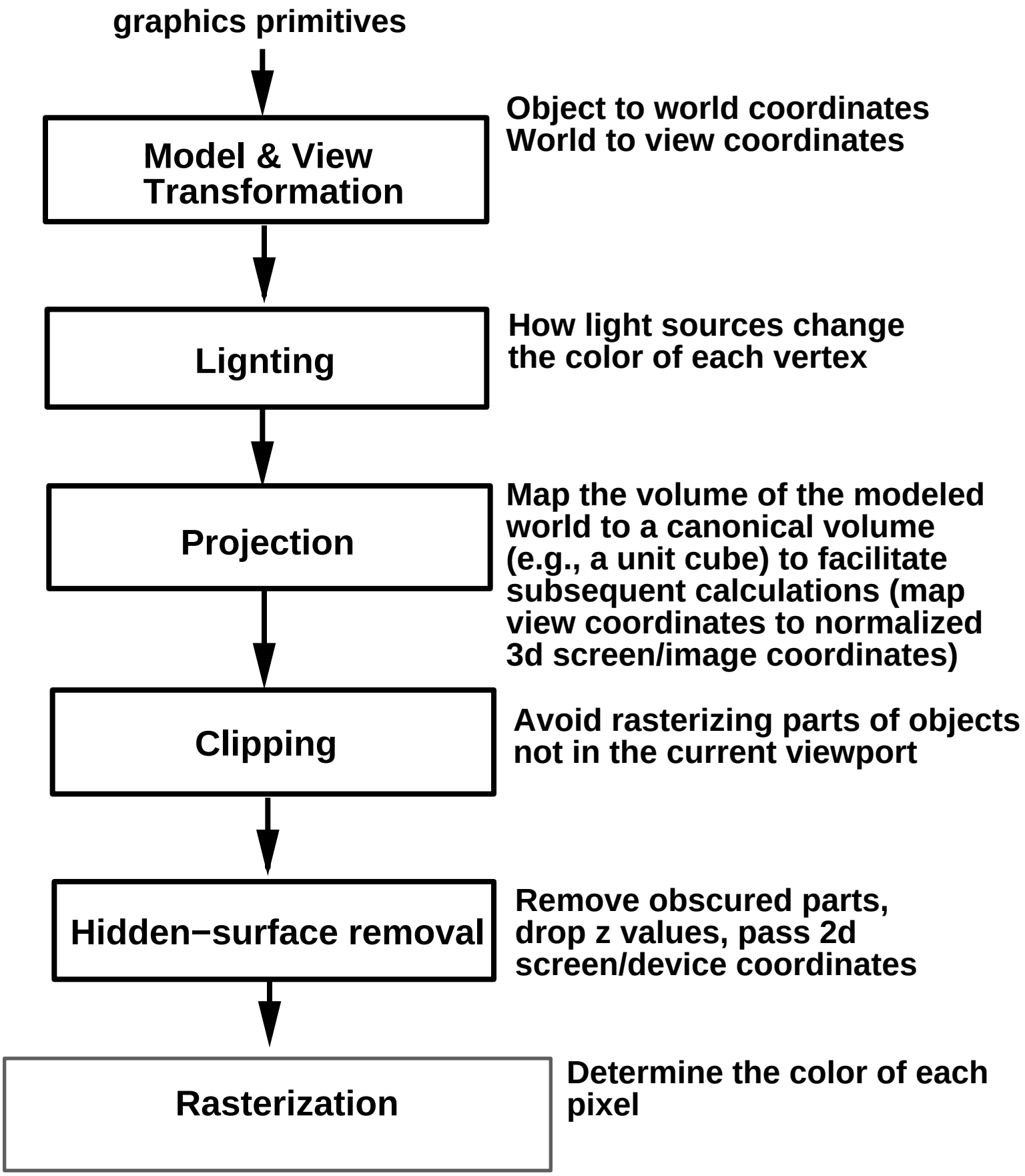
A Computer Graphics Application



Raster Graphics Pipeline



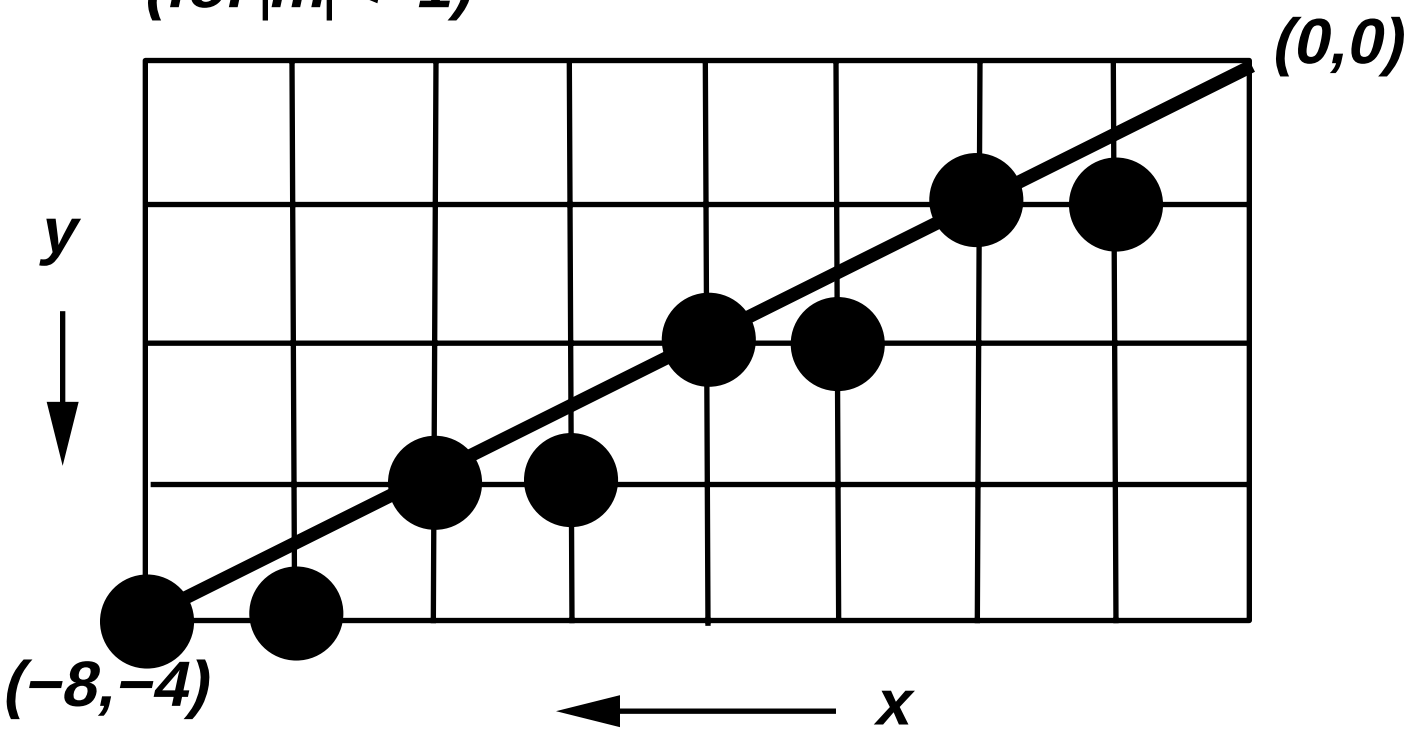
Geometry Stage



DDA − A Simple, Incremental Line Drawing Algorithm

Compute the line slope *m* as $\Delta y/\Delta x$

Increment *x* by 1 and the *y* increment is *m*
(for $|m| < 1$)



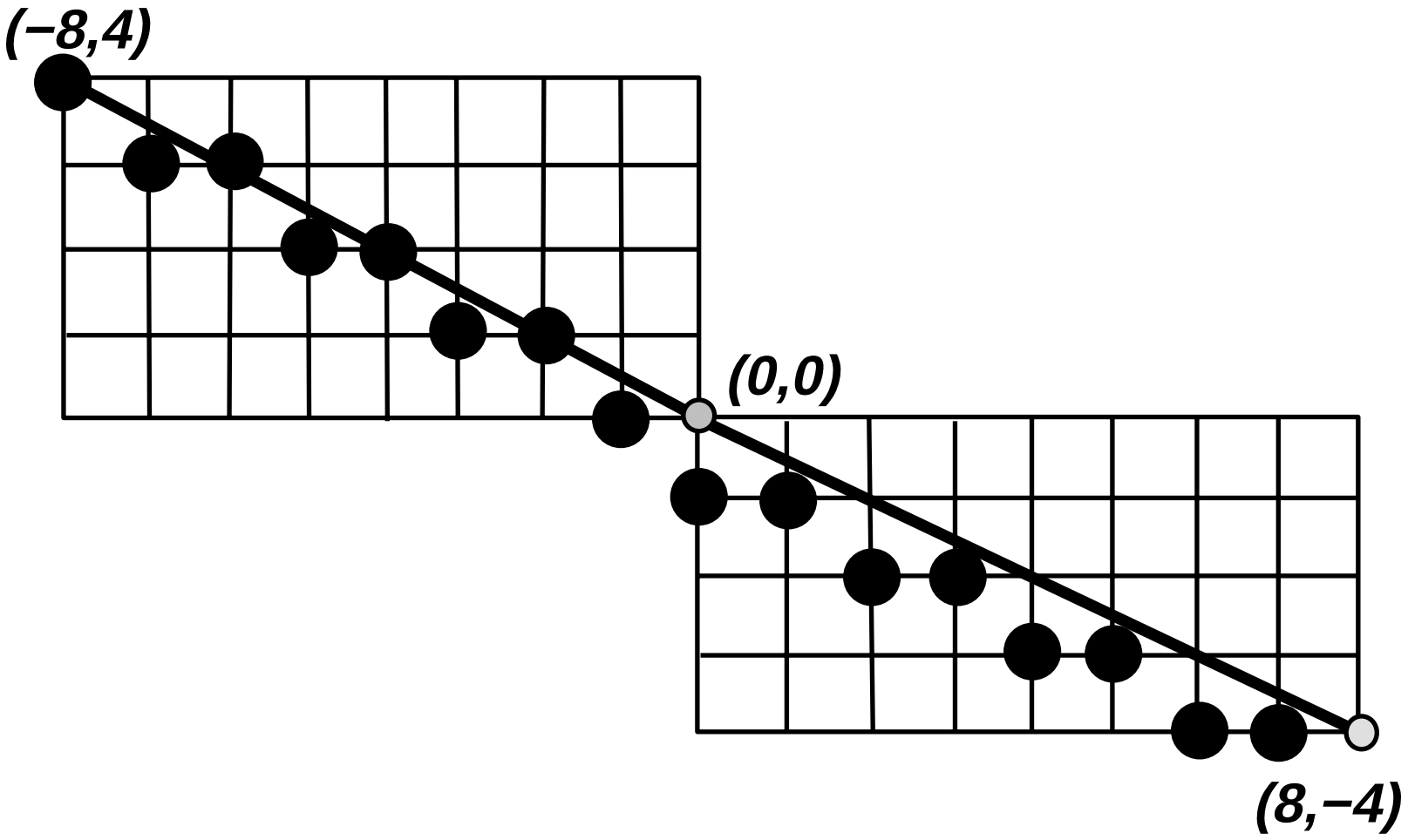
Problems:

1. **Floating point calculations**
2. **Rounding calculations**
3. **Rounding errors**
4. **Orientation dependent**

Rounding Errors

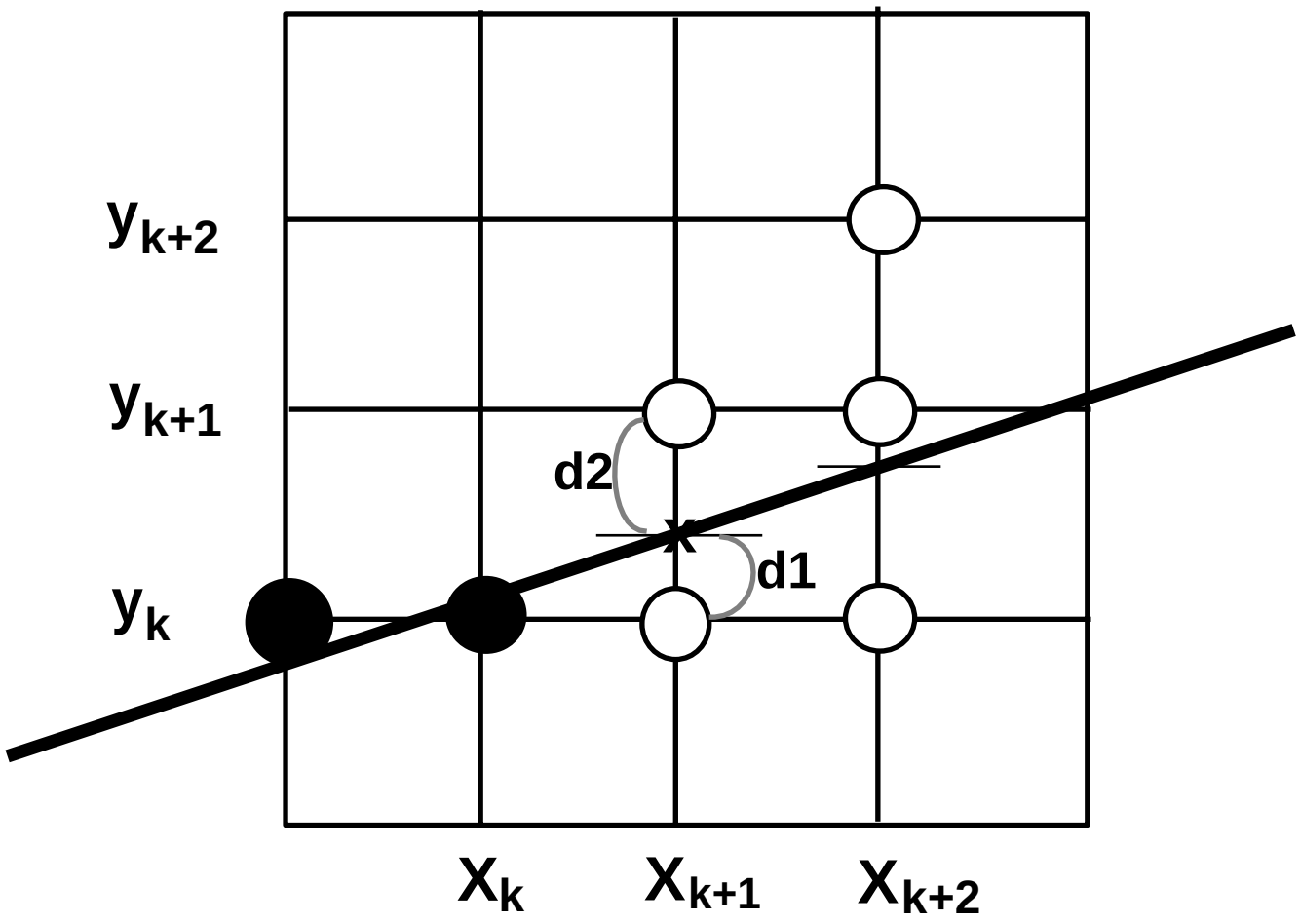
```
Line (integer x0, y0, x1, y1)  {

    integer x;
    float dx, dy, y, m;

    dx = y1−y0;
    dy = x1−x0;

    m = dy/dx;

    y = y0;

    for x = x0 to x1  {

        writePixel(x, Round(y));
        y = y + m;

    }
}
```

Orientation Dependent Problem



Bresenham's Line Algorithm

(0 < m < 1)



d2 > d1 plot	y_k
else Plot	$y_k + 1$

Is it possible to compute and compare d1 and d2 using only integer operations?

$$y = m\,(x_k + 1) + b$$

$$d2 = (y_k + 1) - y = y_k + 1 - m\,(x_k + 1) - b$$

$$d1 = y - y_k = m\,(x_k + 1) + b - y_k$$

$$d1 - d2 = 2m\,(x_k + 1) - 2y_k + 2b - 1$$

$$d1 - d2 = 2\frac{dy}{dx}(x_k + 1) - 2y_k + 2b - 1$$

$$dx(d1 - d2) = 2\,dy\,(x_k + 1) - 2dx\,y_k + dx(2b - 1)$$

let $2dy + dx(2b-1) = c$

We have:

$$dx(d1 - d2) = 2dy \cdot x_k - 2dx\,y_k + c$$

since $0 < m < 1$, $dx > 0$

$dx(d1-d2)$ has the same sign as $(d1-d2)$, so

if $(d1-d2) < 0$, then $y_{k+1} = y_k$

else $y_{k+1} = y_k + 1$

Let $P_k = (d1-d2) = 2dy \cdot x_k - 2dx\, y_k + c$

where $c = 2dy + dx(2b-1)$

$P_{k+1} = 2dy \cdot x_{k+1} - 2dx\, y_{k+1} + c$

To derive P_{k+1} from P_k:

$$P_{k+1} - P_k = 2\,dy\,(x_{k+1} - x_k) - 2dx\,(y_{k+1} - y_k)$$

$$= 2\,dy - 2dx\,(y_{k+1} - y_k)$$

$$P_{k+1} = P_k + 2dy - 2\,dx\,(y_{k+1} - y_k)$$

$$P_0 = 2\,dy \cdot x_0 - 2dx \cdot y_0 + 2dy + dx\,(2y_0 - 2(dy/dx)\,x_0 - 1)$$

$$= 2dy - dx$$

Bresenham's Line Algorithm

Given end points (x_0, y_0) (x_1, y_1)

$dx = x_1 - x_0$, $dy = y_1 - y_0$

Starting with an end point (x_0, y_0):

1. Compute $P_0 = 2dy - dx$
2. For each k, staring with k=0

   if ($P_k < 0$)

      the next point is (X_k+1, Y_k)

      $P_{k+1} = P_k + 2\,dy$

   else

      the next point is (X_k+1, Y_k+1)

      $P_{k+1} = P_k + 2dy - 2dx$

3. Repeat step 2 $x_1 - x_0$ times