<u>Lecture 10: 4/30/2009</u>

Announcements: Ps5 out: **Review Problems, not to be turned in**
Ps4 solutions out; Ps5 Solutions out later today and Sample midterm solutions
Midterm Tues, May 5; Open Book and Notes; Coverage through relations (today)
Discussion this Friday: Review session (run by me). Bring questions

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Functions (Chapter 3)

Definition: A function f is a relation on A x B such that
there is one and only one pair a R b for every a ∈A.

We write b=f(a) to mean that (a,b) ∈f.

(Just one way to do it: we could have defined functions as the primitive and used the function to define the relation, putting in a pair (a,f(a)) for every a ∈A.)

- We call A the **domain** of f, Dom(f).
- We call B the *__codomain__* (or *target*) of f.
  Note that this does **not** mean the set {b: f(a)=b for some a in A}!
  That is a different (and important) st called the *__Range__* (or *image*) of f. Denote it f(A).

Example 1:
Domain={1,2,3}
f(a) = a^2.
Dom(f) = {1,2,3}
f(A) = {1,4,9}
co-domain: unclear, might be \N, might be \R, ...

Example 2:
Domain = students in this class
b(x) = birthdays, encoded as {1,..,12} x {1..31}.

b(phil) = (7,31)
b(ellen) = (4,1)

Example 3:
f: \R -> \R defined by $f(x) = x^2$
is it a function?
Represent it as a graph

Two functions f and g are equal, f=g, if their domains and ranges are equal
and f(x) = g(x) for all x in Dom(f)

Function composition
f o g
f: A -> B,   g: B -> C
then (g o f) : A -> C is defined by
(g o f)(x) = g(f(x))

Kind of "backwards" notation, but fairly traditional. Some algebrists
will reverse it,   (x) (f o g)   "function operates on the left"

Some computer scientists like to denote functions by "lambda expressions"
To say that f is the function that maps x to x^2 we write
f = lambda x. x^2
Here x is just a formal variable;
lambda x . x^2 =   lambda y . y^2
The domain is not explicitly

Functions don't have to be defined on numbers, of course

|x| = maps ∑* -> \N
hd(x) = the first character of the string x, x≠emptystring
tl(x) = all but the first character of x (define how when x=\emptystring)?
dim(A) = the dimensions of the matrix A, regarded as a pair of natural numbers

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2. One-to-one, Onto functions

Def: f:A -> B is *injective* (or *one-to-one*) if
f(x)=f(y) -> x=y

Def: f:A -> B is *surjective* (or *onto*) if   // the image is the co-domain
(for all b in B) (there exists a in A) f(a)=b

Def: f:A -> B is *bijective* (1-to-1 and onto) if
f is injective and surjective.

Example:

-f(x) = x+1 on \N   1-1, not onto
-f(x) = x+1 on \Z   1-1, onto
-f(x) = 2x on \N   1-1, not onto
-f(x) = 2x on \R   1-1, onto
-f(x) = x^2 on \R   not 1-1, not onto
-f(x,y) = x + y on \N x \N   not 1-1, onto
-f(x) = |x| etc....
-f(x) = \lfloor x \rfloor
-f(x) = \lceil x \rceil
-f(x) = x mod n
x = an + b   where a is an integer and b in [0..n-1]

-f: regular expressions -> regular expressions
f(alpha) = lex first regular expression the language of which is alpha,
not 1-1, not onto
-f(M)=1 if M is a DFA and L(M) is infinite, 0 otherwise

Sometimes a little tricky to see if a function is 1-1, onto:

-f(x) = 3x mod 4   bijective
-f(x) = 3x mod 5   not bijective

Make a table

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2. Inverse of a function

If f(x) = y we say that x is a *preimage* of y
Does every point in the codomain have a preimage?
No, only points in the image.
Does every point in the image have *one* preimage?
No, only if it's an injective function
Does every point the in the domain have an image?

Yes, that's required by a function.
Might it have two images?
No, only one.

So if you do have an bijective function f: A -> B,
the function f^{-1}: B -> A is well defined:
f^{-1}(y) = the unique x such that f(x) = y.

Example: f(x) = exp(x) = e^x
Draw picture.
What's the domain? \R
What's the image? (0,∈ f ty)
Is it 1-1 on this image? YES

What's it's inverse

$e^x = y$
$x = \ln(y)$

Example: f(x) = $x e^x = y$ draw it.
On what domain does the inverse exists?
answer: [0,infinity]
Could you define a larger portion of the curve on which the inverse exists:
Yes, [-1/e .. infinity]

Graph the function. Take its derivative to see that it vanishes at -1, when the function takes on the value -1/e.

How would you find the inverse $f^{-1}(2)$, for example?

$x e^x = 2$.
Binary search

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3. Important functions

\lceil x \rceil
lfloor x \rfloor
a mod b
|x|
e^x
2^x
3^x
log
ln
lg
n!

Review of properties of logs:
   log(ab) = log(a) + log(b)
   log_a(b) = log_c(b) / log_c(a)
   $e^{ab} = (e^a)^b$
   $a^x a^y = a^{\{x+y\}}$
Draw picture of y = lg(x)

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4. Comparing functions

Which is bigger, x^3 or 2^x?
Nor formal meaning we know -- it depends on x.
But there is some sense in which 2^x seems bigger
Draw graphs.