## Lecture 15: 5/21/2009

## Announcements: Ps8 out. Ps7, solutions out

Counting, Chapter 5 5.2 Basics (sum, product rule)

inclusion/exclusion counting:  $|A \setminus cup B| = |A| + |B| - |A \setminus cap B|$ (and generalizations).

5.3 Functions: Factorial

n! is well approximated by  $(2\pi n)$  .<sup>5</sup> n<sup>n</sup> /e<sup>n</sup> Stirling's approximation.

10! Is about 3.6 million20! About 2.4 \* 1018 (computed using Google)

C(n,r) also written nCr *for n choose r*. Since if you choose r things from n, n-r are left, C(n,r) = C(n, n-r)

e.g C(10,7) = C(10,3) = 10\*9\*8/(3\*2\*1) = 120

C(n,r) are called Binomial coefficients since they appear in the expansion of  $(a+b)^n$  (see theorem 5.2 in the text: Note typo there, "n choose r" should be "n choose k" Pascal's triangle of coefficients:



Example counting exercises

Please calculate values explicitly; I like to see the numbers.

1. License plates in Nebraska are 3 distinct letters (A-Z, but not O), followed by 3 distinct numbers. How many possible license plates are there?

Answer: 25\*24\*23\*10\*9\*8 = P(25,3) P(10,3) = 9,936,000

2. How many ways can a blue, white, and red ball be put into 10 bins? Assume no bin can contain two balls.

Answer: 10\*9\*8 = P(10,3) = 720

3. How many different orders can a salesman travel among n cities, where he starts in city 1 and visits each other city once and only once before returning to city 1.

Answer: (n-1)!

4. How many ways can you select a president, vice president, and treasurer in a club of 30 people (can't hold two offices)?

Answer: P(30,3) = 24,360

5. How many way can you form Male-Female dance partners if there are 12 women and 8 men. Assume each man is partnered with some woman (4 women go un-partnered).

Answer: P(12,8) = 19,958,400

6. How many ways you position 7 people in a circle?

Answer: 6! = 720

7. A man, a woman, a boy, a girl, a dog, and a cat are walking single-file down the road.

- a. How many ways can this happen? Answer: 6! = 720
- b. How many ways if the dog comes first? Answer: 5! = 120
- c. How many ways if the dog immediately follows the boy? Answer: 5! = 120
- d. How many ways if the dog (and only the dog) is immediately between the man and the boy.Answer: 2\*4! = 48 (form a man-dog-boy or a boy-dog-man combo)
- 8. In how many ways can 10 adults and 5 children be positioned in a line so that no two children are next to each other? (they fight)

Answer: 10!\*P(11,5) = 10! 11! / 6! = 201,180,672,000 \approx 10^11.3

9. How many arrangements are there of the letters a..z such that there are exactly 10 letters between the "a" and the "z"?

Answer:  $15!*P(24,10)*2 = 24!*30 \text{ approx } 1.86*10^{25}$ 

(reasoning: after selecting the AxxxxxxXZ block, treat it as atomic and rearrange with the 14 remaining letters in any of 15! ways. Double to account for AxxxxxXXZ and ZxxxxxXXA possibilities.)

10. You take a group of four people to a Chinese restaurant that has 100 different dishes. All food will be shared among the four of you. How many ways can you order 4 dishes?

Answer: C(100,4) = 100\*99\*98\*97 / (4\*3\*2\*1) = 3,921,225

11. You toss a coin 8 times. How many ways can it land with a total of5 heads?

Answer: C(8,5) = 56(Note this is C(8,3). In general, C(n,r) = C(n,n-r).)

12. How many 6-element subsets are there of the 26 letters, A ... Z ?

Answer C(26,6) = 230,230

And if two letters? C(26,2) = 26\*25/2 = 338. In general, C(n,2) = n(n-1)/2

- 13. An urn contains 15 red, numbered, balls, and 10 white, numbered balls. 5 balls are removed.
- (A) How many different samples are possible? Answer: C(25,5) = 53,130
- (B) How many samples contain only red balls? Answer: C(15,5) = 3003.
- (B') So what is the probability that a random sample will contain only red balls Answer: 3003 / 53,130 \approx 0.005652 (0.5652 %) (about 1 in 18)
- (C) How many samples contains 3 red balls and 2 white balls? Answer: C(15,3) \* C(10,2) = 20,475
- (C') So what's the chance that a random sample will contain 3 red balls and one white ball

Answer: 20,475 / 53,130 \approx 0.3854 (38.54%)

For next time: POKER DEFINITIONS:

straight = 5 consecutive cards: A2345, 23456, 34567, ..., 89TJQ, 9TJQK, TJQKA

royal flush = TJQKA of one suit

straight flush = straight in one suit that is not a royal flush

- four of a kind = four cards of one value, eg., four 9's
- full house = 3 cards of one value, 2 cards of another value

flush = five cards of a single suit

- straight = a straight that is not a royal flush or a straight flush
- three of a kind = three cards of one value, a fourth card of a different value, and a fifth card of a third value

two pairs = two cards of one value, two more cards of a second value, and the remaining card of a third value

one pair = two cards of one value, but not classified above