## Lecture 16: 5/61/2009

Announcements: Ps9 out. Ps8, solutions out tomorrow FINAL EXAM: Monday June 8, 10:30-12:30 106 Olson, Open Book, 1 double sided sheet of paper for notes. Review Session: Fri, 6/5: 12:10-2 146 Olson

POKER DEFINITIONS (mostly not done in class):

straight = 5 consecutive cards: A2345, 23456, 34567, ..., 89TJQ, 9TJQK, TJQKA

royal flush = TJQKA of one suit straight flush = straight in one suit that is not a royal flush four of a kind = four cards of one value, eg., four 9's full house = 3 cards of one value, 2 cards of another value flush = five cards of a single suit straight = a straight that is not a royal flush or a straight flush three of a kind = three cards of one value, a fourth card of a different value, and a fifth card of a third value two pairs = two cards of one value, two more cards of a second value, and the remaining card of a third value one pair = two cards of one value, but not classified above

16. How many poker hands are there?

Answer: C(52,5)=2,598,960

17. How many poker hands are full houses?

Answer: A full house can be identified by a pair, like (J,8), where the first component of the pair is what you have three of, the second component is what you have two of. So there are 13\*12 such pairs. For each there are C(4,3)=4 ways to chose the first component, and C(4,2)=6 ways to choose the second component. So all together there are

13\*12\*4\*6=3744 possible full houses.

The chance of being dealt a full house is therefore  $3744/2598960 \approx .001441$ 

18. How many poker hands are two pairs?

Answer: We can identify 2 pairs as in  $\{J,8\}$ . Note that now the pair is now unordered. There are C(13,2) such sets. For each there are C(4,2) ways to choose the larger card and C(4,2) ways to choose the smaller card. There are now 52-8 remaining cards one can choose as the fifth card. So the total is C(13,2)\*C(4,2)\*C(4,2)\*44 = 123,552.

The chance of being dealt two pairs is therefore  $123,552/2,598,960 \approx .047539$ .

Today: o Probability

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1. Basic definitions / theory

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Schaum's, chapt 7.

DEF: A [\*finite\*] \*probability space\* is a finite set S ("the sample space") together with a function P: S-> [0,1] (the \*probability measure\*) that  $\sum P(x) = 1$ 

for x P S

In general, whenever you hear "probability" make sure that you are clear WHAT is the probability space and WHAT is the event in question.

DEF: Let (S, P) be a probability space. An \*event\* is a subset of S. An \*outcome\* is a point in S.
∩
DEF: Let A be an event of probability space
P(A) = \sum P(a) (used to using Pr, will probability slip)
a ∈A

"The probability of event A"

DEF: The \*uniform\* distribution is the one where P(a) = 1/|S| -- all points are equiprobable.

DEF: Events A and B are \*independent\* if  $P(A \cap B) = P(A) P(B)$ .

DEF: A \*random variable\* is a function X: S -> R

DEF: E[X] = \sum P(s)X(s) // expected value of X ("average value") s in S

DEF  $P(A|B) = P(A \cap B)/P(B)$ 

**Propositions:** 

- P(\emptyset) = 0 // by definition
  P(S) = 1
  P(A) + P(S \ A) = 1
- If A and B are disjoint events (that is, disjoint sets) then  $P(A \cup B) = P(A) + P(B)$
- ("sum bound")  $P(A \cup B) \le P(A) + P(B)$
- In general,  $P(A \cup B) = Pr(A) + Pr(B) - P(A \cap B)$  // inclusion-exclusion principle
- Pr(A) = P(A|B1)P(B1) + P(A|B2)P(B2)
   If B1,B2 disjoint sets whose union is S

- E(X+Y) = E(X) + E(Y) // expectation is linear.

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Eg 1: Dice.

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a You roll a fair die six times:  $S = \{1,2,3,4,5,6\}$ P(1)=P(2)=...=P(6)=1/6

"you roll an even number" is an event.

Event is  $A = \{2,4,6\}$ . P(A) = 3 \* (1/6) = 1/2.

b Pair of dice.

 $S = \{1,2,3,4,5,6\} x \{1,2,3,4,5,6\} o P((a,b)) = 1/36 \text{ for all } (a,b) \text{ in } S$ 

Illustrate independence.

P(red die even and blue die even) = P(left die even) P(blue die even) = (1/2)(1/2) = 1/4

c Pair of dice, what's the chance of rolling an "8"?

Event  $E = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$ 

P(E) = 5/36

Be careful: P(E) = |E|/|S| \*if\* we are assuming the \*uniform\* distribution.

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Eg 2. Poker examples

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POKER DEFINITIONS:

Five cards dealt out. Order irrelevant.

What's the probability space? Sample space has |S| = C(52,5)Probability measure is uniform: P(a) = 1/|S|.

full house = 3 cards of one value, 2 cards of another value two pairs = two cards of one value, two more cards of a second value, the remaining card of a third value

A. How many poker hands are there?

Answer: C(52,5)=2,598,960 =|S|

B. What is the probability of being dealt a full house?

Answer: A full house can be identified by a pair of values, like (J,8), where the first component of the pair is the value of what you have three of and the second component is the value of which you have two. So there are P(13,2) = 13\*12 such pairs.

For each there are C(4,3)=4 ways to chose the first component and C(4,2)=6 ways to choose the second component. So all together there are

P(13,2) C(4,3)C(4,2) / C(52,5)

13\*12\*4\*6=3,744 possible full houses.

The probability of being dealt one is therefore

 $P(13,2) C(4,3)C(4,2) / C(52 = 3,744/2,598,960 \approx .001441$ 

C. What's the probability of being dealt two pairs?

Answer: We can identify 2 pairs as in  $\{J,8\}$ . Note that now the pair is now unordered. There are C(13,2) such sets. For each there are C(4,2) ways to choose the larger card and C(4,2) ways to choose the smaller card. There are now 52-8=44 remaining cards one can choose as the fifth card.

So the total number of hands that are two pairs are:

C(13,2)\*C(4,2)\*C(4,2)\*44 = 123,552.

The chance of being dealt two pairs is therefore  $123,552/2,598,960 \ge .047539$ .

Eg 3: Fair coin

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Flip a fair coin 100 times. What is the probability space?  $S = \{0,1\}^{100}$  $P(s) = 2^{-100}$  for all s in S.

What is the chance of getting exactly 50 out of the 100 coin flips  $C(100,50) 2^{(-100)} \approx 0.07959$