Lecture 19: 6/4/2009

Announcements:.
FINAL EXAM: **NOTE ROOM CHANGE** Monday June 8, 10:30-12:30 **6 Olson**, Open Book, 1 double sided sheet of paper for notes.

OH shifts: Me, 12-1 this Thursday (instead of 1-2)
Nick: 2-4 Friday (after the review session)

Review Session: Friday 12:10-2, 146 Olson
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o Graph theory
  o Hints for the final,!

Graph theory chapter 8
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1. Trees
....

Def: A _**tree**_ is a connected acyclic graph.
Def: A _**leaf (**of a tree) is a vertex of degree one (or zero if G has only one node).
Picture.
3

Proposition: A connected graph is tree iff |E|= |V|-1
Proof: By induction by induction on the number of vertices.

Basis: one vertex, zero edges, so true.

Induction step: Assume all connected graphs with n vertices are trees iff n-1 edges. To show it is also true for connected graphs with n+1 vertices, we do just the only if: assume a tree with n+1 vertices, show we have n edges. Simple idea, the tree must have a leaf x with one neighbor y. So remove x and {x,y} to get a new graph with n vertices and one fewer edge. Easy to show the new graph is a tree (still connected and no cycle), thus we use the induction hypothesis to know it has n-1 edges, and thus the original tree with n+1 vertices had n edges.

6. Colorability
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Def: A graph G = (V,E) is *k-colorable* if we can paint the vertices using "colors" {1,...,k} such that no adjacent vertices have the same color.

Def: A graph is bipartite if it is 2-colorable.   In other words, we can partition V into (V1, V2) such that all edges go between a vertex in V1 and a vertex in V2.

Proposition: There is a simple and efficient algorithm to decide if a graph G is 2-colorable.

Proofs: Just color greedily: start with first color and always color neighbors with a different color. If you succeed, then two colorable, if no, impossible.

Amazing fact: There is no reasonable algorithm known to decide if a graph is 3-colorable.
(Most computer scientists believe that no such algorithm exists.)

7. Planar graphs
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Def: A graph is *planar* if you can draw it in the plane with no crossing edges.

Amazing fact: there is a simple algorithm to decide if a graph is planar (and, when it is, to find an embedding)

The complete graph on 5 vertices is not planar, neither is a complete bipartite graph with 3 vertices in each of the two parts. Non-planar graphs contain one of these two so called “forbidden sub-graphs”.

8. Isomorphic graphs
....

Def: Graphs G=(V,E) and G'=(V',E') are ISOMORPHIC if there exists a bijection pi: V -> V'   s.t. {x,y} in E iff {pi(x),pi(y)} in E.

Amazing fact: there is no efficient algorithm known to decide if two graphs are isomorphic. (Most computer scientists believe that no such algorithm exists.)

Hints for Final,   Goodbye

- It will probably look like the midterm in structure .

- Learn the things that have been asked and answered already: problem sets and their solutions / / midterm / practice exams. I do like to make sure that I repeat some questions with little change: this is a way to reward students who didn't get it before but put in the time to get it now, and a way to deal with the fact that sometimes very clever students are not "fast" enough to solve something highly unfamiliar, and being "fast" is not what I aim to grade.

- Some of you are freshman, so I'll give you my sage advice on studying for this or most other exams: Don't spend too much time doing so, as it's stressful and rarely helps. Instead, you really have to be learning the material well throughout the term. Education is a marathon, not a sprint.

I couldn't really learn much new in the days before. Catching a film is good. So is getting some sleep.

GOOD LUCK and HAPPY TO HAVE MET YOU