1 Lect 5 - April 14, 2009 - ECS 20 - Spring 2009

1.1 Sets: Chapter 1

 $S = \{ dog, cat \} S = \emptyset$

$$\begin{split} N &= \{1, 2, 3, ...\} ---- \text{ some books include 0, some don't } \\ R &= \{x : x \text{ is a real number}\} \\ Z &= \{..., -2, -1, 0, 1, 2, ...\} \\ Q &= \{m/n : m, n \in Z, n \neq 0\} \end{split}$$

What set is this? $A = \{2i + 1 : i \in Z\}$

1.2 CS Applications of Sets

In addition to mathematical language for naming things formally, Sets help define types in programming languages (e.g. integers, reals, characters ...) and form the formal basis for many data base queries.

Sets can be named in many ways. Other ways to name A: ={..., -5, -3, -1, 1, 3, 5, ...} ={x : x is an odd integer} ={ $n : n \in Zand \neg (\exists j \in Z)(2j = n)$ }

Def: S = T iff ($x \in S < -> x \in T$)

|S|= the number of element in S if S is finite, ∞ otherwise Book also uses n(S)~=~|S|~ , but that notation is rarely used. A~=~

$$\begin{split} &\{\{a\},b,\emptyset\}. \ |A|=3 \quad \text{NOTE: sets can contain other sets} \\ &A=\{\emptyset,\{\emptyset\},\{\{\emptyset\}\}\} \quad |A|=3. \end{split}$$

 $\{a, b\} \subset \{a, b, c\} \quad \text{YES, more precise}$ $\{a, b\} \subseteq \{a, b\} \quad \text{YES}$ $\{a, b\} \subseteq \{a, d, e\} \quad \text{NO}$ $\{a, b\} \not \subset \{a, d, e\} \quad \text{YES}$ $\emptyset \subseteq \{a, b, c\} \quad \text{YES (explain)}$ $\{\emptyset\} \subseteq \{a, b, c\} \quad \text{YES}$ $\{\emptyset\} \subseteq \{a, b, c\} \quad \text{YES}$ $\{\emptyset\} \subseteq \{\{emptyset\}\} \quad \text{YES}$

T/F: for all S,
$$\emptyset \subseteq S$$
 : True

NOTE: Sets are always considered to be subsets of an underlying Universal set U. Usually it is clear from context (whether we are discussing numbers, or cars, people, or strings of characters), but technically should be specified.

This also lets us talk about the *complement* of a set S, all the elements of U that are NOT in S.

$$\begin{split} \bar{S} &= \; \{ a \in U | a \not\in S \} \\ S^c \text{ is also used for the complement of } S. \end{split}$$

1.3 UNION

 $A \cup B = \{x : x \in A \text{ or } x \in B\} // \text{ not really very rigorous:} S = Everything *** NO - invalid **$

$$\{a, b\} \cup \{\emptyset, a\} = \{a, b, \emptyset\}$$
$$A \cup \emptyset = A$$

 $\cup_{a \in N} \{a^i : i \in N\} = \{1, 1, \ldots\} \cup \{2, 2^2, 2^3, 2^4 \ldots\} \cup \{3, 3^2, 3^3, \ldots\} \cup \ldots$ "powers of integers" = $\{a^i : a, i \in N\}$

Subtle point: is this a set of **numbers** (elements of N) or **symbols** (pairs of numbers with one raise to the second).

1.4 INTERSECTION

 $\begin{array}{ll} \{1,2,3\} \cap \{2,5,8\} &=& \{2\} \\ \{1,2,3\} \cap \{4,5,8\} &=& \emptyset \\ \{1,2,3\} \cap \emptyset &=& \emptyset \text{ T/F } S \cap \emptyset = \emptyset \text{ True} \end{array}$