Problem Set 1 – Due Tuesday, April 7

Recall that homeworks are due at 3:15 pm and are turned in at 2131 Kemper

1. *Show that n^2 is even if and only if n is even (a fact we used in our proof that $\sqrt{2}$ is irrational).* You may use as facts: i) even integers can be represented as $2i$ for some integer i; ii) odd integers can be represented as $2i + 1$ for some integer i (see page 269 in the text).

2. *How many n-disk legal configurations are there in the Tower of Hanoi problem? A "legal configuration" entails that no disk is larger than a disk beneath it on the same peg. All n disks have different diameters.*

3. Suppose we had a "small" deck of only 30 cards. Would our argument that 5 shuffles are not enough to do a good job of mixing the deck still hold? Justify your answer.

4. *Write down the negation of each of the following statements in clear and concise English. Do not use expressions like "it is not the case that" in your answers. Do not worry about whether the statements are true or false.*

   - *Every integer is divisible by a prime.*
   - *There exists a, b, c such that $(ab)c \neq a(bc)$.*
   - *There exists an infinite set whose proper subsets are all finite.*
   - *There exists a planar graph which cannot be colored with at most four colors.*