Problem Set 3 – Due Tuesday, April 21, 2009; 3:15

1. Complete the following table, answering whether the statement is true (True) or false (False) when the universe of discourse is as indicated.

	$\mathbb{R}$	$\mathbb{Z}$
$\forall x \exists y(2x - y = 0)$		
$\exists y \forall x(2x - y = 0)$		
$\forall x \exists y(x - 2y = 0)$		
$\forall x(x < 10 \rightarrow \forall y(y < x \rightarrow y < 9))$		
$\exists y \exists z(y + z = 100)$		
$\forall x \exists y(y > x \wedge \exists z(y + z = 100))$		

2. Translate the negation of the following statements into formulas of quantification logic, introducing predicates as needed.

   (a) There is someone in the freshman class who doesn't have a roommate.

   (b) Everyone likes someone, but no one likes everyone.

   (c) $(\forall a \in A)(\exists b \in B)(a \in C \leftrightarrow b \in C)$

   (d) $(\forall y > 0)(\exists x)(ax^2 + bx + c = y)$

3. A well-formed formula is said to be in *conjunctive normal form* (CNF) if it is the conjunct (and) of terms where each term is the disjunct (or) of variables or their complements. Convert the following formula into CNF: $\phi = A \wedge (B \leftrightarrow C)$. Can every formula be converted into a logically equivalent one in CNF? Explain your answer (HINT: consider the answer zeros in the table.

4. In a survey of 270 college students, the following data were obtained:

   64 had taken a mathematics course,
   90 had taken a computer science course,
   58 had taken a business course,
   28 had taken both a mathematics and a business course,
   26 had taken both a mathematics and a computer science course,
   22 had taken a computer science and a business course, and
   14 had taken all three types of courses.

   (a) How many students were surveyed who had taken none of the three types of courses?

   (b) Of the students surveyed, how many had taken only a computer science course?

5. Suppose that A, B and C are sets. For each of the following statements either prove it is true or give a counterexample to show that it is false.

   (a) $C \in \mathcal{P}(A) \iff C \subseteq A$

   (b) $A \subseteq B \iff \mathcal{P}(A) \subseteq \mathcal{P}(B)$