Problem Set 5 – NOT TO BE TURNED IN

1. Draw a DFA for $L = \{a_1 \dots a_n :$ each a_i is a bit and $a_i = 1$ for all odd $i\}$. Assume that the empty string ε *is* in L.

2. **(a)** Enumerate the first 6 strings of $L = 00^*11^*$ in lexicographic order.[1]

   (b) Draw a DFA for this language. It should have no more than four states.

   (c) Draw a DFA for L^c, the complement of L relative to the alphabet $\{0, 1\}$. It should be obtained from your solution to (b) by generic means.

   (d) Write a regular expression for L^c, trying to "read this off" of your DFA for (c).

3. Let
$$L = \{x \in \{0,1\}^* : \text{the substring } \texttt{000} \text{ occurs an even number of times in } x\}.$$

   (a) Is $\varepsilon \in L$? Is $\texttt{0000} \in L$?

   (b) Draw a DFA for L. *Hint: states of the DFA might represent things like "I've seen an even number of* `000`*'s so far and the longest strings of* `0`*'s that I've just seen has length 0" (that would be the start state).*

4. Consider the relation $R = \{(1,2),(1,4),(2,3),(2,4),(4,4)\}$ on the set $A = \{1,2,3,4,5\}$. Write down, as sets: (a) the reflexive closure of R,
(b) the symmetric closure of R
(c) the transitive closure of R. , and
(d) the smallest superset of R that is an equivalence relation. Also, give the blocks of this equivalence relation.
I suggest you draw directed graphs to help you see figure out (c)–(d).

5. Determine if the following relations R are (i) reflexive, (ii) symmetric, and (iii) transitive.

   (a) xRy if x and y are people who were born on the same day.

   (b) xRy if x and y are strings that contain a common character.

   (c) xRy if x and y are people and there exists a country C that both have been in.

   (d) xRy if x and y are points in the plane that are equidistant to the origin.

6. For $a, b \in \mathbb{R}$ define $a \sim b$ if $a - b \in \mathbb{Z}$.

   (a) Prove that $\sim$ defines an equivalence relation on $\mathbb{Z}$.

   (b) Describe the equivalence class containing 5. Describe the equivalence class containing 5.5.

7. Use induction to prove that $2 + 4 + \dots (2n-2) + 2n = n(n+1)$ for $n = 1, 2, \dots$.

[1] Recall that the lexicographic order of the strings in L means: all strings in L of length 0; then all strings in L of length 1 (these in dictionary order, say $0 < 1$); then all strings in L of length 2 (these in dictionary order); and so on.