Problem Set 6 – Due Tuesday, May 12, 2009: 3:16

1. Let $f(n) = 2f(n-1) + 1$ for $n > 1$ and $n \in \mathbb{Z}$ Also let $f(1) = 1$. Prove by induction that $f(n) = 2^n - 1$ for all $n \geq 1$ and $n \in \mathbb{Z}$

2. For $a, b \in \mathbb{Z}$ define $a \sim b$ if $a + b$ is even.

   (a) Prove that $\sim$ defines an equivalence relation on $\mathbb{Z}$.

   (b) Describe the equivalence class containing 6. Describe the equivalence class containing 5.

3. Let $f(x) = x \lg x$ (the log being base-2). Compute $f^{-1}(10)$ to at least 4 decimal digits of accuracy. You can do this with the help of a calculator or a short computer program.

4. We consider the following setting: you have a computer that can perform 10^9 operations per second. We list below several functions of n that describe the number of operations a given program takes to solve a problem of size n. You are to determine for each function the largest problem the program could solve in: one second, one minute (60 seconds), and one hour.For example, if $f(n) = 2n$ you can solve a problem of size $10^9/2 = 600 * 10^6$ in one second, and $60 * 10^9/2 = 3 * 10^10$ in one minute (all logs are base 2).

   $f_1(n) = 6n \lg n$
   $f_2(n) = 6n^2 + 3n + 7$
   $f_3(n) = 2^n$
   $f_4(n) = \lg n$
   $f_5(n) = 16n$
   $f_6(n) = n + \lg n$

5. Compute the number of times the following code fragment executes statement S as a function of n.

```
for i = 1 to n do
  for j = 1 to i do
       for m = j to j+9 do
          S
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