## Problem Set 7 – Due Monday, May 19, 2009, 3:15

- 1. For each definition below is it a function?, and give its range.
  - a)  $f(x) = x^2$ , the domain and co-domain of f are the reals.
  - b)  $g(x) = \lceil x \rceil,$  the domain and co-domain of g is the reals.
  - c)  $h(x) = x^{.5}$ , the domain and co-domain of h are the reals.
  - d)  $f \circ g(x)$
- 2. Prove that a function  $f: A \leftarrow B$  is invertible iff f is 1-1 and onto .
- 3. Suppose  $f(x) = \Theta(n^3)$  and  $g(x) = O(n^2)$  What can we say about the following in terms of big O and  $\Theta$  terms?
  - a)  $f(x) \times g(x)$
  - b) f(x) + g(x)
  - c)  $f \circ g(x)$
- 4. Show that the function Fibo(n), which returns the *nth* fibonacci number, is in  $O(2^n)$ . Hint, use induction.
- 5. We proved that the positive integers were equinumerous with all integers (and with the rationals). Now let A be the odd positive integers. Show that A is equinumerous with  $\mathbb{N}$ , and thus that A is equinumerous with  $\mathbb{Q}$ .