

Problem Set 8 – Due Tuesday, May 26, 2009; 3:15

1. [18] Prove that for any integer $n \geq 1$, if x_1, \dots, x_n are distinct real numbers, then no matter how the parentheses are inserted into their sum, the number of additions used to compute the sum is $n - 1$.
2. [8] Find the minimum number of students needed to guarantee that 4 of them were born in the same Month.
3. [7] Now suppose we want to find the minimum number of students needed to guarantee that there are two different months such that (at least) 4 of them were born in each Month. Can we do this? Give a value or show why we can't
4. (a) [10] Given an equal arm balance capable of determining only relative weights of two quantities, and eight coins, all of equal weight except possibly one that may be lighter, explain how to determine if there is a light coin, and how to identify it in just two weighings.
(b) [15] Given an equal arm balance as in (a) and $3^n - 1$ coins, $n \geq 1$, all of equal weight except possibly one that is lighter, show how to determine if there is a light coin and how to identify it with at most n weighings.
5. [18] For $n \geq 1$, let $B(n)$ be the number of ways to express n as the sum of 1s and 2s, taking order into account. Thus $B(4) = 5$ because $4 = 1 + 1 + 1 + 1 = 1 + 1 + 2 = 1 + 2 + 1 = 2 + 1 + 1 = 2 + 2$.
(a) *Compute $B(i)$ for $1 \leq i \leq 5$ by showing all the different ways to write these numbers as above.*
(b) *Find a recursive definition for $B(n)$ and identify this sequence.*
(c) *Compute $B(10)$.*
6. [17] For the Binary Search routine we gave in class, we showed that we only return "not found" when X is not in the array. However, other types of subtle bugs could occur. For example, when we compute Middle , if it is outside the range $1 \dots n$ we can get an error for accessing an index not in the array. Show that this can never occur.
7. [7] Solve the following recurrence relation to within a $\Theta(\cdot)$ result. Assume that $T(n) \in \Theta(1)$ for sufficiently small n . You may use the Master Theorem we stated in class. The recurrence is:
$$T(n) = 5T(n/2) + n^2 \quad n > 1$$
$$T(n) = 1 \quad n \leq 1.$$