Problem Set 9 – Due Tuesday, June 2 3:15 PM, June 2, 2009

In each of the problems below, if you are calculating a number, give it to me explicitly as a decimal number. You can use Google as a calculator, or any other calculator or program.

- 1. (a) (20)In how many ways can ten boys and four girls sit in a row?
 - (b) In how many ways can they sit in a row if the boys are to sit together and the girls are to sit together?
 - (c) In how many ways can they sit in a row if the girls are to sit together?
 - (d) In how many ways can they sit in a row if *just* the girls are to sit together?
- 2. (10)There are 55 students. 6 leave together, then a 7th and finally an 8th. In how many ways can this exodus occur?(how the 6 who leave together are arranged doesn't matter, just which 6 they are
- 3. (10)How many permutations over the letters a, b, c, d, e, f, g contain neither the pattern bge nor the pattern eaf?
- 4. (15) A group of people is comprised of six from the USA, seven from China, and eight from India.
 - (a) In how many ways can a committee of six be formed with two people from each country?
 - (b) In how many ways can a committee of seven be formed with at least two people in each country?
- 5. (25) Suppose we deal five cards from a standard deck of 52 playing cards

a) How many such hands (sets of five cards) are all of the same suit? If you deal out a random hand, what is the probability that it will have this property?

b) How many such hands (sets of five cards) are a straight (A,2,3,4,5; 2,3,4,5,6; ..., T,J,Q,K,A)? If you deal out a random hand, what is the probability that it will have this property?

c) Suppose the first card dealt is a King. What is the probability that the hand will end up being a straight (more precisely, of the hands which start with a king, what fraction will end up being a straight)?

d) Suppose the first card dealt is 7. Now, what is the probability that the hand will end up being a straight?

- 6. (20) Consider the following one-person game. Alice initially has \$300. A dealer has a deck of 52 cards, perfectly shuffled. The dealer turns over the cards one-by-one. If at some point Alice has x then, just before the next card is turned over, Alice can bet any amount of money in [0, x]. The bet is "red" or "black". If the card turned over has the color Alice said, then Alice wins x; otherwise, she loses x.
 - (a) Describe a strategy such that Alice is *guaranteed* to end the game with \$600.
 - (b) Describe a strategy such that Alice is guaranteed to end the game with at least \$800.
 - (c) (Thought problem—not graded—I don't know the answer.) What's the most money that Alice can be certain to win?