Problem Set 5—Due Friday , March 2. 3:15 hardcopy, 11PM electronic

(25) Problem 1. 11-7

(20) Problem 2 10-3

(30) Problem 3. Vertex covers in Bipartite Graphs We will show that we can use network flow/matching to find minimum cardinality vertex covers in bipartite graphs.

a) Argue that for any matching M in an undirected graph G (whether bipartite or not), the minimum cover of G must be at least of size $|M| = k$.

b) Suppose we have a bipartite graph $G = (L, R, E)$, where L is the left vertex set and R the right vertex set. We can form the normal flow network (as discussed in class and in the book) to find a maximum flow f and its associated matching M^* in G. Let S, T be the minimum-s, t-cut associated with f. Prove that $C = (L \cap T) \cup (R \cap S)$ is a minimum cover of G by showing that:

i) for every edge $(u, v) \in M^*$ exactly one of u and v is in C.

ii) for every edge $(u, v) \in E$, At least one of u and v is in C. (Thus C is a cover).

iii) $|C| = |M^*|$ (hint: it may be useful to show that if a vertex x is unmatched, then it is not in C).

(25) Problem 4. In 10.2 we showed that we can find the maximum weight independent set in a tree in linear time using dynamic programming. Now consider a variation of the independent set problem, a *1-independent set*, which is a set S of vertices such at at most one pair of vertices in S is connected by an edge (thus we allow one pair of vertices to be connected by an edge, in contrast to normal IS when we allow zero). We consider the 1-IS problem on trees.

a For the unweighted case, is it still always OK to add a leaf to the solution and obtain a maximum size 1-IS? Justify your answer.

b For the weighted case, modify the DP formulation of 10.2 to efficiently find a maximum weight 1-IS when the input graph is a tree. Justify your answer and give its run time.

(0) Study Problem: not to be turned in 11-10