1 global min-cut computation using $2n$ flows

We wish to show that, given the global min-cut of a graph, (S_{mc}, T_{mc}) and it's capacity $C(S_{mc}, T_{mc})$, that computing the min-cut for any $s \in S_{mc}$ and $t \in T_{mc}$ via network flow results in the same capacity min-cut, $C(S_{mc}, T_{mc})$. Note that this result applies to any flow network, not just to ones with unit capacity arcs (though the time to do each flow will be longer for a general flow graph).

- **claim** Given (S_{mc}, T_{mc}), and fixing any pair $s \in S_{mc}$ and $t \in T_{mc}$, the min-cut computed with max network flow from s to t has the same capacity (noting there may be multiple min-cuts with the same value, so a min-cut is not necessarily unique).

  pf:

  The min-cut from s to t cannot be less then $C(S_{mc}, T_{mc})$ since this our assumed global min-cut. Then, the capacity must be $\geq C(S_{mc}, T_{mc})$. Assume it is greater (otherwise done). But the min $s - t$ capacity is the min capacity set of edges to disconnect s from t. But in (S_{mc}, T_{mc}), this chosen s IS disconnected from t (if fact every node in S_{mc} is disconnected from t). Thus, (S_{mc}, T_{mc}) is a valid $s - t$ cut, and thus the max-flow from s to t and the min-cut cannot be greater than $C(S_{mc}, T_{mc})$. Since the min $s - t$ cut value cannot be smaller or greater than $C(S_{mc}, T_{mc})$, it must be equal to it.

  Then, the claim has been proven. We can take any $s \in S_{mc}$ and any $t \in T_{mc}$ and the capacity of this min-cut is the capacity of the global min-cut, $C(S_{mc}, T_{mc})$.

  This then allows us to compute the global min-cut of a graph by fixing one node v as the source, computing max-flow/min-cut to all sinks, then fixing v as the sink, and computing a flow for all sources. By the proof above, we can claim that at least one of these min-cuts will be the global min-cut value.