Software Alchemy:

Turning the Complex into Embarrassingly Parallel

Norm Matloff

Department of Computer Science, University of California at Davis

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On the Web

This PDF file contains my presentation at the R meeting. I've extended the document by including material summarizing the question-and-answer period of that talk, and will occasionally add some updates as well.

The most up-to-date version of these slides, and associated R code, will be available on the Web at

http://heather.cs.ucdavis.edu/barugApr11/.

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• **"Solution":** Run in parallel only if you have an embarrassingly parallel algorithm. :-)

A (Rather) General Method

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- I will present a rather general solution here...I might even say a panacea.
- Works for most *statistical* problems.
- Our goal here: Turn highly NON-EP problems into EP ones!

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- Old, old idea in parallel processing: Break data into chunks, work on each chunk, then combine results.
- But this requires EP.
- New approach: Exploit the <u>statistical</u> properties.
- Key point: Calculate a **statistically equivalent** quantity that lends itself to EP computation.

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- Have n data points, p processes (e.g. p = 2 for dual core on a single machine).
- Break into r chunks of n/p data points each.
- For i = 1,...,r calculate $\hat{\theta}$ on chunk i, yielding $\tilde{\theta}_i$.
- Average all those chunked values:

$$\overline{\theta} = \frac{1}{r} \sum_{i=1}^{r} \widetilde{\theta}_i$$

What Does That Give You?

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- Alchemy! Non-EP \rightarrow EP.

Example: Regression

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What chunking does here:



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- will have the same statistical accuracy, but will be faster

Some Experiments with Regression

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- $\bullet\,$ used 3 dual-core PCs, so $p\leq 6$
- regression is a non-EP problem

Elapsed times in seconds (single runs):



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n	q	р	ordinary	NM method	gputools
500000	30	6	4.18	3.58	8.40
500000	50	6	9.41	6.61	exceeded mem.
100000	100	6	4.13	3.55	3.86
50000	150	6	4.14	3.36	2.92

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NM method "handicapped": used **snow** (which uses **serialize()**), over a network.

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Second Example: Quantile Regression

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• Model the population conditional quantiles, say medians, as a linear function.

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Second Example: Quantile Regression

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Elapsed times in seconds (single runs):

n	q	р	ordinary	NM method
10000	50	2	2.39	1.50
10000	50	4	2.39	1.34
50000	50	4	36.10	13.43
50000	50	6	35.51	11.19

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Question: Does this only work on linear regression problems?

- No, the math works on any function of i.i.d. data.
- I've tried it on logistic regression, principle components and estimation of hazard functions from censored data, getting modest to excellent speedups.
- Note that if $\widehat{\theta}$ is an unbiased estimator, then $\overline{\theta}$ is also unbiased.

Question: Is there a convergence rate issue in your asymptotics?

- In my experiments I've found only tiny differences between $\overline{\theta}$ and $\widehat{\theta}$.
- The only problems that are worth parallelizing have very large sample sizes, and thus the asymptotics have certainly taken effect by then.