Name: ____________________________

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing.

1. X be the number of dots we get in rolling a three-sided die once. (It's cylindrical in shape.) The die is weighted so that the probabilities of one, two and three dots are 1/2, 1/3 and 1/6, respectively. Note: Express all answers in this problem as common fractions, reduced to lowest terms, such as 2/3 and 9/7.

(a) (10) State the value of $p_X(2)$.

(b) (10) Find EX and Var(X).

(c) (15) Suppose you win $2 for each dot. Find EW, where W is the amount you win.

2. This problem concerns the **REVISED** version of the committee/gender example.

(a) (10) Find $E(D^2)$. Express your answer as an *unsimpilfied* expression involving combinatorial quantities such as $\binom{168}{28}$.

(b) (15) Find $P(G_1 = G_2 = 1)$. Express your answer as a common fraction.

3. (15) State the (approximate) return value for the function below, in terms of w. **You must cite an equation number in the book to get full credit.**

```r
xsim <- function(nreps,w) {
   sumn <- 0
   for (i in 1:nreps) {
      n <- 0
      while (TRUE) {
         n <- n + 1
         u <- runif(1)
         if (u < w) break
      }
      sumn <- sumn + n
   }
   return(sumn/nreps)
}
```

4. (15) Consider the parking space example on p.48. (NOT the variant in the homework.) Let N denote the number of empty spaces in the first block. State the value of Var(N), expressed as a common fraction.

5. (10) Suppose X and Y are independent, with variances 1 and 2, respectively. Find the value of c that minimizes Var[cX + (1-c)Y].

Solutions:

1.a 1/3

1.b

$$EX = 1 \cdot (1/2) + 2 \cdot (1/3) + 3 \cdot (1/6) = 5/3$$

$$\begin{aligned} Var(X) &= E(X^2) - (EX)^2 \\ &= 1^2 \cdot (1/2) + 2^2 \cdot (1/3) + 3^2 \cdot (1/6) - 25/9 \\ &= 5/9 \end{aligned}$$

1.c EW = E(2X) = 2 EX = 10/3

2.a

$$E(D^2) = (-2)^2 \frac{\binom{6}{1}\binom{3}{3}}{\binom{9}{4}} + ...$$

2.b

$$\begin{aligned} P(G_1 = G_2 = 1) &= P(G_1 = 1)P(G_2 = 1 | G_1 = 1) \\ &= \frac{6}{9} \cdot \frac{5}{8} \\ &= \frac{5}{12} \end{aligned}$$

3. 1/w, by (3.74)

4. 10(0.2)(1-0.2) = 8/5, by (3.82)

5.

$$\begin{aligned} 0 &= \frac{d}{dc} Var[cX + (1-c)Y] \\ &= \frac{d}{dc} \left[c^2 Var(X) + (1-c)^2 Var(Y)\right] \\ &= \frac{d}{dc} [c^2 + 2(1-c)^2] \\ &= 2c - 4(1-c) \end{aligned}$$

So, the best c is 2/3.