

Name: _____

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing.

1. (25) Consider the simple board game, pp.15ff, starting at 0. Change the game so that it has 16 squares, numbered 0 to 15, but is otherwise identical to the original one. Let X denote the square the player lands on after the first turn. Find $E(X)$, expressing your answer as a sum of fractions, e.g. $3/2 + 1/5$ ($-3/8$).

2. (25) Suppose we have a random variable X , and define a new random variable Y , which is equal to X if $X > 8$ and equal to 0 otherwise. Assume X takes on only a finite number of values (just a mathematical nicety, not really an issue). Which one of the following is true:

- (i) $EY \leq EX$.
- (ii) $EY \geq EX$.
- (iii) Either of EY and EX could be larger than the other, depending on the situation.
- (iv) EY is undefined.

3. This problem concerns the binary tree model in our homework.

(a) (25) Find the probability that the root has exactly 1 grandchild, expressing your answer in terms of p , algebraically simplified.

(b) (25) Fill in the blanks in the following code simulating the function $r(k,p)$:

```
simltree <- function(k,p) {
  if (k == 0) return(          ) # blank
  prevlevelbranches <- 1
  for (m in 1:          ) { # blank
    newbranches <- 0
    for (i in 1:          ) { # blank
      for (j in 1:2) {
        if (sample(0:1,1,prob=c(1-p,p)) == 1) {
          newbranches <- newbranches + 1
        }
      }
    }
  }
  if (newbranches == 0) return(0) # blank
}
return(1)
}
```

```
treesim <- function(p,k,nreps) {
  count <- 0
  for (i in 1:nreps) {
    treetok <- simltree(k,p)
    count <- count + treetok
  }
  return(          ) # blank
}
```

Solutions:

1. X can take on the values 1 through 9. Then

$$EX = 1 \cdot P(X = 1) + 2 \cdot P(X = 2) + \dots + 9 \cdot P(X = 9) \tag{1}$$

$P(X = 1) = \frac{1}{6}$, $P(X = 4) = \frac{1}{6} + \frac{1}{36}$, etc.

2. Answer (iii) is correct. If we'd had the additional condition $X \geq 0$, then (i) would have been right. But without that condition, then for instance suppose X were always negative; then Y would always be larger, etc.

3.a The root will have exactly one grandchild iff it has two children, and one of them has one child and the other has none. Thus the queried probability is

$$p^2 \cdot [2p(1-p)] = 2p^3(1-p) \tag{2}$$

3.b

```
simltree <- function(p,k) {
  if (k == 0) return(1)
  prevlevelbranches <- 1
  for (m in 1:k) { # levels
    newbranches <- 0
    for (i in 1:prevlevelbranches) {
      for (j in 1:2) { # account for left , right outlinks
        if (sample(0:1,1,prob=c(1-p,p)) == 1) {
          newbranches <- newbranches + 1
        }
      }
    }
    if (newbranches == 0) return(0)
    prevlevelbranches <- newbranches
  }
  return(1)
}

treesim <- function(p,k,nreps) {
  count <- 0
  for (i in 1:nreps) {
    treetok <- simltree(p,k)
    count <- count + treetok
  }
  return(count/nreps)
}
```