Name: ____________________________

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing.

1. (20) Consider the ALOHA example, Sec. 3.14.3. Write a call to the built-in R function **dbinom()** to evaluate (3.123) for general m and p.

2. Consider the bus ridership example, Sec. 2.10. Suppose upon arrival to a certain stop, there are 2 passengers. Let A denote the number of them who choose to alight at that stop.

(a) (10) State the parametric family that the distribution of A belongs to.

(b) (20) Find $p_A(1)$ and $F_A(1)$, writing each answer in decimal expression form e.g. $12^8 \cdot 0.32 + 0.3333$.

3. (20) Consider the following simple inventory model. A store has 1 or 2 customers for a certain item each day, with probabilities p and q (p+1 = 1). Each customer is allowed to buy only 1 item.

When the stock on hand reaches 0 on a day, it is replenished to r items immediately after the store closes that day.

If at the start of a day the stock is only 1 item and 2 customers wish to buy the item, only one customer will complete the purchase, and the other customer will leave emptyhanded.

Let X_n be the stock on hand at the end of day n (*after* replenishment, if any). Then $X_1, X_2, ...$ form a Markov chain, with state space 1,2,...,r.

Write a function **inventory(p,q,r)** that returns the π vector for this Markov chain. It will call **findpi1()**, similarly to the two code snippets in p.65.

Solutions:

1.

```
dbinom(1,m,p)
```

2.a binomial

2.b

$$p_A(1) = 2(0.2)(0.8) \tag{1}$$

$$F_A(1) = P(A = 0 \text{ or } A = 1) = (0.8)^2 + 2(0.2)(0.8) \tag{2}$$

3.

```
inventory <- function(p,q,r) {
   tm <- matrix(rep(0,r^2),nrow=r)
   for (i in 3:r) {
      tm[i,i-1] <- p
      tm[i,i-2] <- q
   }
   tm[2,1] <- p
   tm[2,r] <- q
   tm[1,r] <- 1
   return(findpi1(tm))
}
```