Name: ____________________________

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing.

SHOW YOUR WORK!

1. (15) Suppose $f_X(t) = 1/t^2$ on $(1, \infty)$, 0 elsewhere. Find $h_X(2.0)$, expressing your answer as a common fraction, reduced to lowest terms.

2. (10) The book, *Last Man Standing*, author D. McDonald writes the following about the practice of combining many mortgage loans into a single package sold to investors:

> Even if every single [loan] in the [package] had a 30 percent risk of default, the thinking went, the odds that most of them would default at once were arguably infinitesimal...What [this argument] missed was the auto-synchronous relationship of many loans...[If several of them] are all mortgage for houses sitting next to each other on a beach...one strong hurricane and the [loan package] would be decimated.

Fill in the blank with a term from our course: The author is referring to an unwarranted assumption of ____________________________________.

3. (15) In the light bulb example on p.188, suppose the actual observed value of $\overline{X}$ turns out to be 15.88. Write R code to find the p-value. (If you are not sure of the parameters for some R function, just assume what they are and explain what you did.)

4. Suppose X and Y are independent geometrically distributed random variables with success probability p. Let Z = X/Y. We are interested in EZ.

(a) (20) Due to independence, EZ = (1/p) E(1/Y). Find an infinite series expression for E(1/Y).

(b) (20) Write a full R function **findez(nreps,p)** that finds EZ by simulation. The return value is EZ (which of course is only approximate). Extra Credit will be given for the most compact code.

(c) (20) Find an infinite series expression for $F_Z(m)$ for positive integer m. For full credit, there should be only one series. (Extra Credit for a series-free expression.)

Solutions:

1.

$$
\begin{aligned}
h_X(2.0) &= \frac{f_X(2.0)}{1 - F_X(2.0)} && (1) \\
&= \frac{f_X(2.0)}{1 - \int_1^{2.0} f_X(t)\ dt} && (2) \\
&= (1/4.0)/(1 - 1/2.0) && (3) \\
&= 1/2 && (4)
\end{aligned}
$$

2. independence

3. Setting w in Equation (6.65) to 15.88, we need to find what the 0.05 in that equation would change to. The R code

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1 - pgamma(15.88,10,0.001)
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does this.

4.a Using our mailing tube E, p.35, as usual, together with the pmf for the geometric family, we have

$$E(\frac{1}{Y}) = \sum_{i=1}^{\infty} \frac{1}{i}(1-p)^{i-1}p$$

4.b

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findez(nreps,p) return(mean(rgeom(nreps,p)/rgeom(nreps,p)))
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4.c

$$
\begin{aligned}
F_Z(m) &= P\left(\frac{X}{Y} \leq m\right) && (5)\\
&= P(X \leq mY) && (6)\\
&= \sum_{i=1}^{\infty} P(X=i)\ P(X \leq mY | X = i) && (7)\\
&= \sum_{i=1}^{\infty} (1-p)^{i-1} p\ P(Y \geq mi) && (8)\\
&= \sum_{i=1}^{\infty} (1-p)^{i-1} p\ (1-p)^{mi-1} && (9)\\
& && (10)
\end{aligned}
$$

This last expression can be simplified:

$$
\begin{aligned}
F_Z(m) &= \sum_{i=1}^{\infty} (1-p)^{i-1} p\ (1-p)^{mi-1} && (11)\\
&= \frac{p}{(1-p)^2} \sum_{i=1}^{\infty} (1-p)^{(m+1)i} && (12)\\
&= \frac{p}{(1-p)^2} \sum_{i=1}^{\infty} [(1-p)^{m+1}]^i && (13)\\
&= \frac{p}{(1-p)^2} (1-p)^{m+1} \sum_{j=0}^{\infty} [(1-p)^{m+1}]^j && (14)\\
&= \frac{p}{(1-p)^2} (1-p)^{m+1} \frac{1}{1-(1-p)^{m+1}} \quad (3.69) && (15)\\
&= p(1-p)^{m-1} \frac{1}{1-(1-p)^{m+1}} && (16)
\end{aligned}
$$