GROUP QUIZ SUBMISSION INSTRUCTIONS:

- Your work must be submitted by 12 noon, March 12. Submission must be done from within the classroom.
- Submit your work on **handin**, to the directory **132quiz8**.
- Your **.tar** file name must conform to the rules explained in our Syllabus, Section 19.4.
- Your **.tar** file must comprise three files, named **Problem1a.R**, **FindEta.R** and **Problem 2.R**, with contents as specified below.

My grading script will be

```
source("Problem1a.R")
whicheqn()
# set p (not shown)
source("FindEta.R")
findeta(p)
source("Problem2.R")
# set c, n, nreps (not shown)
cmp2ests(c,n,nreps)
```

You are welcome to search the Web, though my saying this should not be construed to mean you necessarily would benefit from this.

1. Consider an n-state Markov chain that is *irreducible*, meaning that it is possible to get from any state i to any other state j in some number of steps. Define the random variable T_{ij} to be the time needed to go from state i to state j. (Note that T_{ii} is NOT 0, though it can be 1 if $p_{ii} > 0$.)

$$E(T_{ij}) = \sum_k p_{ik} E(T_{ij} \mid W = k), \quad 1 \le i,j \le n \tag{1}$$

where W is the state traveled to immediately after leaving state i. This then implies that

$$E(T_{ij}) = 1 + \sum_{k \ne j} p_{ik} E(T_{kj}), \quad 1 \le i,j \le n \tag{2}$$

We'll focus on the case $j = n$, i.e. look at how long it takes to get to state n. Let η_i denote $E(T_{in})$, and define $\eta = (\eta_1, \eta_2, ..., \eta_{n-1})'$. (Note that η has only $n-1$ components!) So,

$$\eta_i = 1 + \sum_{k \ne j} p_{ik} \eta_k, \quad 1 \le i,j \le n \tag{3}$$

In this problem you'll develop an R function to calculate η.

Here is a easy (though trivial) example of η. Suppose the transition matrix of the chain is

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \tag{4}$$

Then one can see right away without any computation that

$$\eta = (2,1)' \tag{5}$$

(a) Give the number of our textbook equation that most justifies (1), among material *prior* to Chapter 4.

Your answer will take the form of an R function **whicheqn()** that consists of a single **print()** call, e.g.

```
print("(2,168)")
```

(b) Using (2), write an R function with call form

```
findeta(p)
```

that inputs the Markov chain's transition matrix **p** and returns η.

Hints: Remember, in (3) the p_{ij} are the knowns, and the η_i are the unknowns. Start with a very simple example, say (4).

2. This problem concerns the raffle example in Section 13.1 of our book. We have two competing estimators, and you will write simulation code to compare them in terms of bias and mean absolute error,

$$b1 = E(\widehat{c}) - c, \quad b2 = E(\check{c}) - c,$$

$$e1 = E(|\widehat{c} - c|), \quad e2 = E(|\check{c} - c|)$$

Your code will consist of a function with call form

```
cmp2ests(c,n,nreps)
```

and will return the (approximate) vector **c(b1,b2,e1,e2)**. Here **c** and **n** are as in the raffle example (but are general, unrelated to the specific data in that example), and **nreps** is our usual number of "notebook lines." Assume sampling with*out* replacement, even though the theory behind $\check{c}$ is based on with-replacement sampling.[1]

[1] Thus assume that $n < c$.

Solutions:

1a. (3.154)

1b.

```
findeta <- function(p) {
   n <- nrow(p)
   q <- diag(n) - p
   q <- q[1:(n-1),1:(n-1)]
   ones <- rep(1,n-1)
   solve(q,ones)
}
```

2.

```
cmp2ests <- function(c,n,nreps) {
   out <- matrix(nrow=nreps,ncol=2)
   for (i in 1:nreps) {
      x <- sample(1:c,n,replace=FALSE)
      out[i,1] <- 2*mean(x) - 1
      out[i,2] <- max(x)
   }
   c(
      mean(out[,1] - c),
      mean(out[,2] - c),
      mean(abs(out[,1] - c)),
      mean(abs(out[,2] - c)))
}
```