Revisiting the Available Cases Method for Missing Values

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JSM 2015

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Forgotten method:

- Available cases (AC). Use partially-intact cases when possible.

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Overview of AC Method

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- Same for the quantities $E[X^{(i)}X^{(j)}]$.

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- For MI, we use Amelia 2.

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- Verdict: Use CC.

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- Simulation example: (1)(23) model, n = 100, est. p_{111}.

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AC advantage more if have more factors or higher NA %.

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