Sampling Theorem

Signals Revisited

- Real world signals contain many different frequencies
- Consider an orchestra. The sound you hear is the signal
  - The tuba makes deep, low sounds. These are low frequency signals.
  - The piccolo or flute makes high pitch notes. These are high frequency.
  - The music you hear is the combination of all of these frequencies.

Signals Revisited

- Images also have frequencies
  - An area of rapidly changing texture, e.g. sand paper or narrow stripes, would be high frequency
  - An area where the color changes gradually would be low frequency

Sampling Theorem

- How many samples do you need to take of a signal in order to be able to accurately reconstruct it?
- Assume that we know the maximum frequency in the signal
  - (a big assumption)
- Fourier series represents signals as a sum of sine waves (beyond the scope of this course)

Sampling Theorem

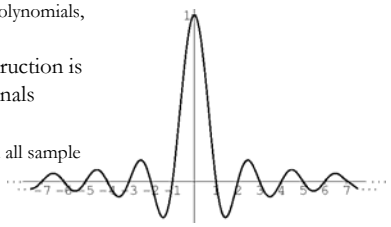
- For a sine wave, frequency is the number of periods or cycles per unit of time
- For sin t
  - frequency = $\omega = 1/2\pi$
    - When t goes from 0 to 2 π, this will complete one full sine wave (an "up" lobe and a "down" lobe)
    - 2π is 360 degrees
    - The period of the sine wave is 2π seconds
    - The frequency is 1/period or $1/2\pi$

Sampling Theorem

- Sampling Theorem: sampling rate must exceed $2\omega_m$
  - ω_m is the max frequency
  - $2\omega_m$ is called the Nyquist Sampling Rate
- If sample rate is lower, signal is *undersampled*
  - Cannot reconstruct original signal
- More than $2\omega_m$ means the function is *oversampled*
  - Often useful in practice as a non-ideal reconstruction function may be used

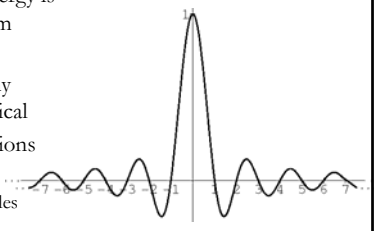
Reconstruction [Advanced Concept]

- Many ways to reconstruct function from samples
  - e.g. fit lines to them, fit higher order polynomials, etc.
- Only one construction is ideal for 1D signals
  - sinc function
  - Convolve with all sample points



Reconstruction

- sinc function
  - Convolve with all sample points
- Problematic as energy is significant far from center
- Must use too many points to be practical
- Use simpler functions (e.g. B-splines)
  - Need more samples



Undersampled

- Samples from an undersampled signal represent a different signal
  - Called an *alias*
- Alias is generally a bad approximation to original signal
- What should be done if you cannot obtain enough samples?
  - Low-pass filter the original signal to a max frequency you can sample

- Look at example of sine wave of different frequencies

Key Points

- Number of samples needed depends on the highest frequency in the signal
- Need at least $2\omega_m$ samples
- For most real reconstruction techniques, you need more
- Can low pass the original signal before sampling at a lower rate
  - Less error in reconstruction