ECS 120 Final — Spring 2004

Hints for success:

Please read the questions carefully; maybe they ask something different from what you expect. If you don't understand what a question means, ask.

Make your writing legible, logical, and succinct. Definitions and theorem statements should be complete and rigorous.

Final grades should be ready by June 20. I will post them to the web.

Have a great summer. Do something interesting! —Phil Rogaway

Name:

Signature:

On problem	you got	out of
1		27
2		26
3		20
4		20
5		20
Σ		113

1 Short Answers [27 points]

---

1. *Complete the following definition*:

A context free language L is **inherently ambiguous** if ...

---

2. *Complete the following definition*:

A language A polynomial time reduces to a language B, written $A \leq_{\mathrm{P}} B$, if

---

3. Using the procedure given in class and in the book (the "subset construction"), convert the following NFA into a DFA for the same language.

4. Give a CFG with a minimum number of rules for the language:

$$L = \{xx^R : \quad x \in \{a,b\}^*\}$$

---

5. Prove that the following language is *not* context free.

$$L = \{xx : \quad x \in \{a,b\}^*\}$$

---

6. Give a (deterministic) procedure to **accept**

$L = \{\langle M \rangle : M$ is a TM with alphabet $\Sigma = \{0,1\}$ that accepts some palindrome$\}$.

7. Describe a procedure that **decides**

$L = \{\langle G \rangle : G = (V, \Sigma, R, S) \text{ is a CFG and } L(G) = \Sigma\}$

---

8. Draw a DFA with a minimal number of states that accepts $L = \{x \in \{1,2\}^* : x \text{ has exactly two 2's}\}$

---

9. If $x \in \{\mathtt{0},\mathtt{1}\}^*$ let x^c denote the bitwise complement of x (so $1001^c = 0110$, for example). For $L \subseteq \{\mathtt{0},\mathtt{1}\}^*$, let $L^c = \{x^c : x \in L\}$. Given an n-state NFA M_1 and an n-state NFA M_2, what is the smallest NFA you can provide for $(L(M_1) \cup L(M_2))^c$?

2 True or False [26 points]

Put an **X** through the **correct** box. No justification is required.

1. If Π is decidable then $\Pi \leq_{\mathrm{m}} \{0,1\}$. **True** **False**

---

2. Every subset of a DFA-acceptable language is DFA-acceptable.
True **False**

---

3. $L^* = (L^*)^*$. **True** **False**

---

4. If G is a CFG grammar in CNF (Chomsky Normal Form) and string x has an m-step derivation under G, and x also has a different, n-step derivation under G, then $m = n$. **True** **False**

---

5. The language $L = \{1^{a_1}\#1^{a_2}\#\cdots\#1^{a_n} : a_i = a_{i+1} \text{ for some } 1 \leq i < n\}$ is context free. **True** **False**

---

6. If L is context free and $\overline{L}$ is also context free then L is regular.
True **False**

---

7. There are infinitely many languages over the alphabet $\Sigma = \{1\}$ for which $L = L^*$.
True **False**

---

8. The class P is closed under complement. **True** **False**

---

9. $\{\langle G, w\rangle : G \text{ is a CFG and } w \in L(G)\} \in P$. **True** **False**

---

10. Context-free languages are closed under intersection. **True** **False**

---

11. The r.e. languages are closed under intersection. **True** **False**

---

12. Every subset of a regular language is regular. **True** **False**

---

13. Let $M = (Q, \{0,1\}, \delta, q_0, F)$ be a DFA and suppose $\delta^*(q_0, x) = \delta^*(q_0, y)$. Then $x \in L(M)$ iff $y \in L(M)$. **True** **False**

3 Language Classification [20 points]

Classify as:

- **decidable** — decidable (recursive)
- **r.e.** — Turing-acceptable (recursively enumerable) but not decidable
- **co-r.e.** — co-Turing-acceptable but not decidable
- **neither** — neither Turing-acceptable nor co-Turing-acceptable

No explanation is required.

---

1. $\{\langle M\rangle : M \text{ is a TM and } L(M) \text{ is finite}\}$ ☐

2. $\{d : \text{the digit } d \text{ appears infinitely often in the decimal expansion of } \pi = 3.14159\cdots\}$ ☐

3. $\{\langle G\rangle : G \text{ is a CFG and } L(G) = \{0,1\}^*\}$ ☐

4. $\{\langle \alpha\rangle : \alpha \text{ is an regular expression and } L(\alpha) = \{0,1\}^*\}$ ☐

5. $\{\langle M\rangle : M \text{ is a TM and } M \text{ accepts some palindrome}\}$ ☐

6. An undecidable language L for which $L \leq_{\mathrm{m}} A_{\mathrm{TM}}$. ☐

7. $\{\langle P\rangle : P \text{ is a C-program and } P \text{ halts on input of itself}\}$. ☐

8. $\{\langle M, M'\rangle : M \text{ and } M' \text{ are TMs and } L(M) = L(M')\}$. ☐

9. $\{\langle G\rangle : G = (V, \Sigma, R, S) \text{ is a CFG and } L(G) = \Sigma^*\}$. ☐

10. $\{\phi : \phi \text{ is a satisfiable Boolean Formula}\}$ ☐

4 Mapping Reductions [20 points]

Let $L = \{\langle M \rangle : M \text{ is a TM and } L(M) = \{\varepsilon\}\}$.

Part A. Prove that L is *not* r.e.

Part b. Prove that L is *not* co-r.e.

5 NP-Completeness [20 points]

Let $BOTH = \{\langle\phi\rangle : \phi$ is a Boolean formula and ϕ has some satisfying truth assignment t_1 and some non-satisfying truth assignment $t_0\}$.

Is *BOTH* NP-Complete? Prove your answer.